MATHEMATICAL SCIENCES

PAPER - II

Signature of Invigilators

1. ....

2. ....

Roll No. ☐☐☐☐☐

(In figures as in Admit Card)

JY–06/01

Roll No. ....

....

(in words)

Name of the Areas/Section (if any)...............

Time Allowed :75 Minutes] **[Maximum Marks : 100**

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page.
2. This paper consists of **Seventy Four (74)** multiple choice type questions. **Attempt any fifty (50)** questions.
3. Each item has upto four alternative responses marked (A), (B), (C) and (D). The answer should be a capital letter for the selected option. The answer letter should entirely be contained within the corresponding square.

   Correct method [A] Wrong method [A] OR [A]

4. Your responses to the items for this paper are to be indicated on the ICR Answer Sheet under Paper II only.
5. Read instructions given inside carefully.
6. Extra sheet is attached at the end of the booklet for rough work.
7. You should return the test booklet to the invigilator at the end of paper and should not carry any paper with you outside the examination hall.

પરીક્ષાર્થીઓ માટે સૂચનાઓ :

૧. આ પાનાની ટોચમાં દર્શાવેલી જગ્યામાં તમારો રોલ નંબર લખો.

૨. આ પ્રશ્નપત્રમાં કુલ **ચુમ્મોતેર (૭૪)** બહુવૈકલ્પિક ઉત્તરો ધરાવતા પ્રશ્નો આપેલા છે. **કોઈપણ (૫૦)** પ્રશ્નોના જવાબ આપો.

૩. પ્રત્યેક પ્રશ્ન વધુમાં વધુ ચાર બહુવૈકલ્પિક ઉત્તરો ધરાવે છે. જે (A), (B), (C) and (D). વડે દર્શાવવામાં આવ્યા છે. પ્રશ્નનો ઉત્તર કેપીટલ સંજ્ઞા વડે આપવાનો રહેશે. ઉત્તરની સંજ્ઞા આપેલ ખાનામાં બરાબર સમાઈ જાય તે રીતે લખવાની રહેશે.

   ખરી રીત : [A] ખોટી રીત : [A] , [A]

૪. આ પ્રશ્નપત્રના જવાબ આપેલ ICR Answer Sheet ના Paper II વિભાગની નીચે આપેલ ખાનાઓમાં આપવાના રહેશે.

૫. અંદર આપેલ સૂચનાઓ કાળજીપૂર્વક વાંચો.

૬. આ બુકલેટની પાછળ આપેલું પાનું રફ કામ માટે છે.

૭. પરીક્ષા સમય પૂરો થઈ ગયા પછી આ બુકલેટ જે તે નિરીક્ષકને સોંપી દેવી. કોઈપણ કાગળ પરીક્ષા ખંડની બહાર લઈ જવો નહીં.

MATHEMATICAL SCIENCES
Paper - II

NOTE : This paper contains SEVENTY FOUR (74) multiple-choice / Assertion & Reasoning/ Matching questions, each questions carrying two (02) marks. Attempt ANY FIFTY questions.

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1. Let $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots\ldots\ldots\ldots\ldots\ldots + \frac{1}{2n}$ $(n = 1,2,3,\ldots\ldots)$

   Then

   (A) $\{a_n\}$ is not bounded

   (B) $\{a_n\}$ is bounded but not convergent

   (C) $\{a_n\}$ is convergent and $\lim_{n \to \infty} a_n \geq \frac{1}{2}$

   (D) $\{a_n\}$ is convergent and $\lim_{n \to \infty} a_n < \frac{1}{2}$.

2. Define $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

   as a function from IR to IR.

   Then

   (A) f is continuous at all points except $x = 0$

   (B) f is continuous at all irrationals only

   (C) f is continuous at $x = 0$ only

   (D) f is continuous at all rationals only.

3. Let $f : \mathrm{IR} \to \mathrm{IR}$ and $g : \mathrm{IR} \to \mathrm{IR}$ be two functions such that f.g (product) is differentiable at 0. Then

   (A) both f and g must be differentiable at 0

   (B) exactly one of f and g must be differentiable at 0, and $f(0)\, g(0) \neq 0$

   (C) neither f nor g need be differentiable at 0

   (D) one of f and g must be differentiable at 0 and both f(0) and g(0) are 0.

4. Let $f\left(\frac{1}{n}\right) = -1 \quad (n = 1, 2, 3, \ldots\ldots)$

$$f(x) = 0 \quad \left(x \in [0,1], x \neq \frac{1}{n}\right).$$

Then

(A) f is continuous on [0, 1]

(B) f is not continuous but is Riemann integrable on [0, 1]

(C) f is not Riemann integrable on [0, 1] but $|f|$ is Riemann integrable on [0, 1]

(D) f is Riemann integrable on [0, 1] but $|f|$ is not.

5. Let $f(z) = -2(xy + x) + i(x^2 - 2y - y^2)$.

(A) f is not differentiable at 0

(B) f is differentiable at 0 but not analytic at 0

(C) f is analytic at all points of $\mathbb{C}$ but is not bound in $\mathbb{C}$

(D) f is analytic and bounded in the whole complex plane $\mathbb{C}$.

6. The series $\sum_{n=2}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}$ for $|z| < \infty$ represents

(A) $\sin z - z$

(B) $\tan z$

(C) $\sec z - \frac{z^3}{3!}$

(D) $\cos z - 1 + \frac{z^2}{2}$.

7. If C is the unit circle taken in the positive direction $\int_C \frac{1}{z}\, dz =$ ________

(A) $2\pi i$

(B) 0

(C) $-2\pi i$

(D) 1.

8. The dimension of the space of all linear maps from $\mathbb{R}^n$ to $\mathbb{R}^n$ (for $n \geq 2$ is

(A) n

(B) n !

(C) n^2

(D) infinite.

9. The coefficient of λ^3 in the characteristic polynomial $f(\lambda)$ of $\begin{pmatrix} 2 & 3 & 0 & 1 \\ 4 & -1 & 0 & 0 \\ 0 & 3 & -4 & 8 \\ 2 & 1 & -4 & 2 \end{pmatrix}$ is

(A) 2
(B) 3
(C) 0
(D) 1.

10. Let $T = \mathbb{R}^2 \to \mathbb{R}^2$ be defined by T (x, y) = (2x – 5y, x – 2y). What are the invariant subspaces of T ?

(A) only { 0 }
(B) infinitely many
(C) $\mathbb{R}^2$ and { 0 }
(D) $\mathbb{R}^2$, { 0 } and the subspace generated by (1, – 1).

11. Which of the following statements is false ?

(A) A 3 x 3 matrix with only one eigen value cannot be diagonalised.
(B) A 3 x 3 matrix with two eigen values cannot be diagonalised
(C) A 3 x 3 matrix is diagonalizable iff it has 3 distinct eigen vectors.
(D) A 3 x 3 matrix must have three distinct eigen values to be diagnotizable.

12. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ which one of the following matrices is the zero matrix ?

(A) $A^2 + 2A + 3I$
(B) $A^2 - 3A + 4I$
(C) $A^2 - 5A - 2I$
(D) $A^2 - 2A + 4I$.

13. Let X be a binomial r.v. with mean $\frac{4}{3}$ and variance $\frac{8}{9}$. Then P(X = 2) is

(A) $\frac{1}{3}$
(B) $\frac{8}{9}$
(C) $\frac{8}{27}$
(D) $\frac{1}{27}$.

14. Let $F(x) = \begin{cases} 0 & \text{if} \quad x < 1 \\ (x+1)/3 & \text{if} \quad 1 \le x < 2 \\ 1 & \text{if} \quad x \ge 2 \end{cases}$

be the d.f. of a r.v.X. then E (X) is

(A) $\frac{4}{3}$
(B) $\frac{1}{2}$
(C) $\frac{1}{3}$
(D) $\frac{7}{6}$.

15. Let a sample space S be the open interval (0, 1). Let events A, B and C be given by the intervals (0.6, 0.9), (0.7, 1) and (0, 0.5) respectively. If the probability of an event is equal to the length of the interval, which of the following statements is FALSE ?

(A) $P(A \cup B) = 0.4$

(B) $P(A \cap B) = 0.2$

(C) $P(B \cup C) = 0.8$

(D) events B and C are mutually exclusive and independent.

16. A card is drawn at random from an ordinary pack of 52 cards and is replaced. This is done 5 times. What is the conditional probability of drawing the ace of spades exactly 4 times given that the ace is drawn atleast 4 times ?

(A) $\frac{51}{52}$

(B) $\frac{255}{256}$

(C) $\frac{12}{13}$

(D) $\frac{1}{2}$.

17. E and F are independent events such that the probability of atleast one of them occurs is $\frac{1}{3}$ and the probability that E occurs but F does not occur is $\frac{1}{9}$. Then P(F) is

(A) $\frac{4}{9}$

(B) $\frac{1}{3}$

(C) $\frac{1}{9}$

(D) $\frac{2}{9}$.

18. X is a Poisson r.v. with variance equal to 2.5. Which of the following statements is FALSE ?

(A) $E(X^2) = 8.75$

(B) coefficient of variation of X is $\frac{1}{\sqrt{2.5}}$

(C) $P(X > 0) = 1 - P(X \leq 0)$

(D) $\frac{P(X=1)}{P(X=0)} = \sqrt{2.5}$.

19. Let X be a closed Convex subset of the Euclidean space E^n and let $y \in E^n$ Consider the statements.

S1 : $y \in X$

S2 : there exists a hyperplane containing y such that all $x \in X$ belong to one open half space produced by that hyperplane.

Then which of the following is correct ?

(A) only S1 (B) only S2

(C) either S1 or S2 (D) neither S1 nor S2.

20. Consider the LPP consisting of m constraints in n variables $(n \geq m)$ for which the following statements are made by a student.

S1 : A basic feasible solution (BFS) has at least $(n - m)$ zeros

S2 : A BFS is degenerate if any of its basic variable is zero

S3 : A BFS has exactly $(n - m)$ zeros.

Then

(A) only S1 is incorrect (B) only S2 is incorrect

(C) only S3 is incorrect (D) only S3 is correct.

21. Let A be an uncountable subset of IR. Then

(A) A may not have any limit point

(B) A has atleast one limit point

(C) A has a limit point only if it is bounded

(D) every element of A is its limit point.

22. Let IR and Q have usual topology and $f : IR \to Q$ be continuous (IR = the set of reals, Q = the set of rationals). Then

(A) $f(x) = x \ \forall x \in IR$ (B) f is one-to-one

(C) f is onto Q (D) f is a constant map.

23. Let f and g be continuous maps from IR to IR. Then

(A) max (f, g) is continuous but min (f, g) may not be

(B) min (f, g) is continuous but max (f, g) may not be

(C) none of max (f, g) and min (f, g) is continuous

(D) both max (f, g) and min (f, g) are continuous.

24. Let f, g, h be real valued functions on [0, 1]. Suppose that f is continuous on [0, 1], g is continuously differentiable on [0, 1] and h is decreasing on [0, 1]. Which of the following is necessarily true ?

(A) f and g are of bounded variation on [0, 1]

(B) only h is of bounded variation on [0, 1]

(C) g and h are of bounded variation on [0, 1]

(D) f and h are bounded variation on [0, 1].

25. Consider IR with a metric d, defined by $d(x, y) = 1$ if $x \neq y$

$= 0$ if $x = y$.

Then for $a < b$

(A) all closed intervals [a, b] are compact

(B) all closed intervals [a, b] are connected

(C) no closed interval [a, b] is compact

(D) every closed subset of [a, b] is compact.

26. The stereographic projection is

(A) a homeomorphism of the complex plane onto a sphere

(B) a one-to-one continuous map of the complex plane into the sphere

(C) a continuous map of the complex plane onto the sphere which is not one-to-one

(D) a one-to-one map of the extended complex plane onto a sphere which is not continuous.

27. Let D be the open unit disc $\{z \in \mathbb{C} \mid |z| < 1\}$ and $f = u + iv$ be a map from D to $\mathbb{C}$. Consider the statements :

p : f is analytic at 0

q : u, v satisfy C-R equations at 0

r : u_x, u_y, v_x, v_y are continuous at 0.

Then

(A) $p \Rightarrow q$ and $p \Rightarrow r$

(B) $p \Rightarrow q$ but $p \not\Rightarrow r$

(C) $q \Rightarrow p$

(D) $q \Rightarrow r$.

28. The image of $\left\{z \in \mathbb{C} \mid \text{Re } z > \frac{1}{2}\right\}$ under the map $w = \frac{1}{z}$ is

(A) $\left\{w \in \mathbb{C} \mid \text{Re } w > \frac{1}{2}\right\}$

(B) $\{w \in \mathbb{C} \mid |w-1| > 1\}$

(C) $\left\{w \in \mathbb{C} \mid |w-1| < \frac{1}{2}\right\}$

(D) $\{w \in \mathbb{C} \mid |w-1| < 1\}$.

29. Let C be the circle of radius 5 centered at 0 and taken in the anticlockwise direction. Then $\int_C \frac{1}{(z-3)(z-6)} dz =$ ____________

(A) $-\frac{2\pi i}{3}$

(B) $\frac{2\pi i}{3}$

(C) $-2\pi i$

(D) 1.

30. A mapping f from a ring R onto a ring R' is called a homomorphism of R onto R' if

(A) $f(a+b) = f(a) + (b);\ f(ab) = bf(a),\ \forall a,\ b \in R$

(B) $f(a+b) = f(b) + f(a);\ f(ab) = af(b),\ \forall a,\ b \in R$

(C) $f(a+b) = f(a) + f(b);\ f(a-b) = f(a) - f(b),\ \forall a,\ b \in R$

(D) $f(a+b) = f(a) + f(b);\ f(ab) = f(a)\, f(b),\ \forall a,\ b \in R$.

31. Under the relation $T(\alpha x + \beta y) = \alpha T(x) + \beta T(y),\ x \in U,\ y \in V,\ \alpha,\ \beta \in F$, we call it a

(A) vector space homomorphism

(B) linear operator

(C) zero transformation

(D) negative transformation.

32. Which one of the following is true ?

(A) every cyclic group is necessarily abelian

(B) every cyclic group is commutative

(C) every cyclic group is a generator

(D) none of these.

33. Two finite dimensional vector spaces U(F) and V(F) are isomorphic if and only if

(A) dim (–U) = dim (V)   (B) $U(F) \cong V(F)$

(C) dim (U) = dim (V)   (D) dim (U) = dim (–V).

34. "Every matrix is zero of its characteristic polynomial". This statement is

(A) linear functional theorem   (B) characteristic theorem

(C) inner product theorem   (D) Cayley-Hamilton theorem.

35. Let $V = \{A \in M_n(IR) : A \text{ is symmetric}\}$. Then dimension of V as a vector space over IR is

(A) n   (B) n^2

(C) $\frac{n(n+1)}{2}$ (D) $\frac{n(n-1)}{2}$.

36. Let $A = \begin{bmatrix} 0 & 1 & -2 & 3 & -4 \\ -1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ -3 & 0 & 0 & 0 & -1 \\ 4 & 0 & -1 & 1 & 0 \end{bmatrix}$ Then

(A) det A = 0   (B) det A = 1

(C) det A = 3   (D) det A = 5.

37. If V is n-dimensional vector space, then the dimension of the space of all linear functionals on V is

(A) n^2 (B) n

(C) $\frac{n(n+1)}{2}$ (D) none of these.

38. If $x_1 = (1, 0, 2)$, $x_2 = (-1, 1, 0)$ and $x_3 = (3, -1, 4)$. Then (0, 1, 2) can be expressed as

(A) Linear combination of x_1 and x_2 only

(B) The linear combination of x_1 and x_2 as well as linear combination of x_1 and x_3.

(C) Linean combination of x_1 and x_3 only

(D) none of these.

39. With usual notations employed, the equation of the envelope being a solution of the differential equation $f(x, y, p) = 0$, its general solution

(A) $\phi(x, y) = 0$ forms the family of curves

(B) $\phi(x, y, c) = 1$ forms the family of lines

(C) $\phi(x, y, c) = 0$ forms the family of lines

(D) $\phi(x, y, c) = 0$ forms the family of curves.

40. In the partial differential equation, Pp + Qp = R, where u and v are solutions of the equation, P and Q are

(A) $\frac{\partial u}{\partial y} \cdot \frac{\partial v}{\partial z} - \frac{\partial u}{\partial y} \cdot \frac{\partial v}{\partial x}; \frac{\partial u}{\partial z} \cdot \frac{\partial v}{\partial x} - \frac{\partial u}{\partial x} \cdot \frac{\partial u}{\partial y}$

(B) $\frac{\partial u}{\partial y} \cdot \frac{\partial v}{\partial z} - \frac{\partial u}{\partial z} \cdot \frac{\partial v}{\partial y}; \frac{\partial u}{\partial z} \cdot \frac{\partial v}{\partial x} - \frac{\partial u}{\partial x} \cdot \frac{\partial v}{\partial z}$

(C) $\frac{\partial u}{\partial x} \cdot \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \cdot \frac{\partial v}{\partial x}; \frac{\partial u}{\partial y} \cdot \frac{\partial v}{\partial y} - \frac{\partial v}{\partial x} \cdot \frac{\partial u}{\partial x}$

(D) None of these.

41. The integrating factor of $\frac{\partial u}{\partial y} + Px = Q$ is

(A) $e^{\int P\,dx}$

(B) $e^{P\int dx}$

(C) $e^{\int Q\,dy}$

(D) $e^{\int P\,dy}$

42. y = x, which is solution of $2\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 2y = 5 + 2x$, is called

(A) complete solution
(B) particular integral
(C) complementary function
(D) singular solution.

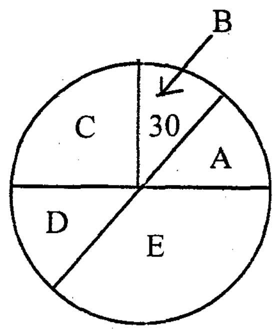
43. The equation Pp + Qp = R is called

(A) partial differential equation
(B) Charpit's linear equation
(C) Lagrange's equation
(D) Lagrange's linear equation.

44. Let f (x, y, z, p, q) = 0 be a non linear partial differential equation. It can be solved by

(A) Lagrange's method
(B) Monge's method
(C) Charpit's method
(D) Separation of Variable method.

45. The profits for the products A, B, C, D and E of a company are indicated by the following pie-diagram.



If the profit for E is Rs. 30,000 more than that of C then profit due to A is

(A) Rs. 45,000
(B) Rs. 60,000
(C) Rs. 30,000
(D) Rs. 90,000.

46. If X + Y and X − Y are uncorrelated then

(A) X and Y are independent
(B) X − Y and X + Y are independent
(C) X and Y have equal means
(D) X and Y have equal variances.

47. Let P_1 and P_2 be probability measures on $(\Omega, \mathcal{A})$. Then

(A) $\alpha P_1 + (1-\alpha) P_2$ is a probability measure for any real number α

(B) $\alpha P_1 + (1-\alpha) P_2$ is a probability measure only for $\alpha = \frac{1}{2}$.

(C) $P(A) = P_1(A) \cdot P_2(A)$ is a probability measure

(D) none of (A), (B), (C).

48. Let X have the d.f. $F(t) = \begin{cases} 0 & \text{if} \quad t < -1 \\ (2+t)/4 & \quad -1 \le t < 1 \\ 1 & \quad t \ge 1 \end{cases}$

Then var (X) is

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) 1

(D) $\frac{2}{3}$.

49. Let $\rho_{x,y}$ denote the correlation coefficient between X and Y with common variance. Then $\rho_{x,(x+y)}$ is

(A) $\sqrt{\frac{1+\rho_{x,y}}{2}}$

(B) $\sqrt{1+\rho_{x,y}}$

(C) $\sqrt{1+2\rho_{x,y}}$

(D) $\sqrt{\frac{1-\rho_{x,y}}{2}}$

50. Let X_1, X_2, X_3 be a random sample of size 3 from exponential distribution with mean 1. Let $X_{(2)}$ denote the second order statistic. The $E(X_{(2)})$ is

(A) $\frac{1}{2}$

(B) $\frac{3}{4}$

(C) $\frac{5}{6}$

(D) 1.

51. Suppose we know that the number of items produced in a factory during a week is a random variable with mean 500 and variance 100. Then the probability that this week's production will be between 400 to 600 is atlest

(A) .95
(B) .5
(C) .99
(D) .75.

52. Let X be a random variable with d.f.

$$F(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ 1 - e^{-x} & \text{for } x > 0 \end{cases}.$$

Then the value of $P\left(3 \leq e^{X} \leq 4\right)$ is

(A) $\frac{1}{12}$
(B) $\frac{1}{3}$
(C) $\frac{1}{4}$
(D) $e^{-3} - e^{-4}$.

53. Let $X_{(1)} < X_{(2)} < \ldots\ldots < X_{(6)}$ be order statistics from a random sample of size 6 drawn from a distribution with p.d.f.

$$f(x) = \begin{cases} 2x & , 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Then $E[X_{(6)}]$ is

(A) $\frac{2}{3}$
(B) $\frac{5}{6}$
(C) $\frac{6}{7}$
(D) $\frac{12}{13}$.

54. X is distributed as normal with mean 1 and variance 4. Given that

$$\frac{X^2}{4} - \frac{X}{2} + \frac{K}{4}$$

is distributed as chi-square with 1 d.f., the value of K is

(A) 0
(B) 1
(C) 4
(D) $\sqrt{2}$.

55. Let X_1, X_2, X_3 and X_4 be independent standard normal r.v.s. The distribution of

$$\frac{X_1 + X_4}{\sqrt{X_2^2 + X_3^2}}$$ is same as that of aY where

(A) a = 1 and Y has a t distribution with 1 d.f.

(B) $a = \frac{1}{\sqrt{2}}$ and Y has a t distribution with 2 d.f.

(C) $a = \sqrt{2}$ and Y has a t distribution with 2 d.f.

(D) a = 1 and Y has a t distribution with 2 d.f.

56. Let r.v. Y be distributed as uniform over (0, 1). For Y = y let the conditional distribution of X given Y = y be u(0, y). Then E (X) is

(A) $\frac{y}{2}$ (B) $\frac{1}{9}$

(C) $\frac{1}{6}$ (D) $\frac{1}{4}$.

57. Let T_n be an unbiased and consistent estimator of θ and g be any continuous function of θ. Consider the statements

α : $g(T_n)$ is unbiased for $g(\theta)$

β : $g(T_n)$ is consistent for $g(\theta)$.

Then

(A) only α is true (B) only β is true

(C) α and β are both true (D) α and β are both not true.

58. Let X_1, X_2 ...., X_n be i.i.d random variables from N (θ, θ) distribution. A sufficient statistic for θ is

(A) Σx_i^2 (B) Σx_i

(C) $\Sigma |x_i|$ (D) $(\Sigma x_i, \Sigma x_i^2)$.

59. Let $x_1, x_2 \ldots\ldots x_n$ be a random sample from $f_\theta(x) = \frac{1}{\theta}$, $K\theta < x < (K+1)\theta, K > 0$ is known. Then MLE of θ is

(A) $\frac{x_{(1)}}{K}$

(B) $x_{(n)}$

(C) $\frac{x_{(n)}}{K+1}$

(D) $\frac{x_{(1)} + x_{(n)}}{2}$.

60. Let $x_1, x_2 \ldots\ldots, x_n$ be i.i.d exponential r.v.s with rate λ. Let $H_o : \lambda = \lambda_o$ and $H_1 : \lambda = \lambda_1 > \lambda_o$. Then the critical region of the Most Powerful test is of the form

(A) $\overline{x}_n \geq K$

(B) $\overline{x}_n \leq K$

(C) $K_1 \leq \overline{x}_n \leq K_2$

(D) $\overline{x}_n \leq K_1$ or $\overline{x}_n \geq K_2$.

61. To test $H_0 : \theta = 1$ against $H_1 : \theta > 1$, based on a single observation from exponential distribution, with mean θ, a test rejects H_0 if $x > 2$. The size of the test is

(A) $1 - e^{-2}$

(B) $\frac{e^{-2}}{2}$

(C) e^{-2}

(D) $1 - \frac{1}{2} e^{-2}$.

62. Let $x_1, \ldots\ldots, x_n$ be independent $N(0, \sigma^2)$. the acceptance region of MP test for $H_o : \sigma^2 = 1$ against $H_1 : \sigma^2 = 4$ is of the form

(A) $\Sigma x_i^2 \leq C$

(B) $\Sigma x_i^2 \geq C$

(C) $\Sigma (xi - \overline{x})^2 \geq C$

(D) different from the above.

63. Let $X_1, \ldots\ldots, X_n$ be i.i.d U (0, θ) rvs. For $d > 0$ which of the following confidence intervals has maximum confidence level ?

(A) $\left(x_{(1)}, x_{(1)} + d\right)$

(B) $\left(\bar{x} - \frac{d}{2}, \bar{x} + \frac{d}{2}\right)$

(C) $\left(x_{(n)}, x_{(n)} + d\right)$

(D) $\left(x_{(n)} - d, x_{(n)}\right)$.

64. The error degrees of freedom in a two-way analysis of variance table with h rows, k columns and r observations per cell, is

(A) hk - 1

(B) r (h – 1) (k – 1)

(C) hk (r – 1)

(D) (h – 1) (k – 1) (r – 1).

65. Consider the single period inventory model with no set up cost, holding cost h, storage cost p, unit purchase cost c and random demand D. Then the optimal order quantity y satisfies

(A) $P(D \le y) = \frac{c - p}{p + h}$

(B) $P(D \le y) = \frac{p - c}{p + h}$

(C) $P(D \ge y) = \frac{c - p}{p + h}$

(D) $P(D \ge y) = \frac{p - c}{p + h}$.

66. The standard (s, S) inventory policy refers to the situation where

(A) the stock level q is expected to satisfy $s \le q \le S$ at all times

(B) $q \ge s$ always but may exceed S

(C) $q \le S$ always but may fall short of s.

(D) the stockist is advised to place order if q < S.

67. Consider a queueing system (M/M/1) which is in a steady state. If the birth rate $\lambda_n = \lambda$ and the death rate $\mu_n = \mu$, then the number of customers in the system at any time follows :

(A) geometric dist over {0, 1, 2,....} if $\lambda < \mu$

(B) geometric distribution over {1, 2, 3, ...] if $\lambda < \mu$

(C) geometric distribution over { 0, 1, 2,.... }

(D) geometric distribution over {1, 2, 3, ... }.

68. Consider the LLP to maximize $Z = 3x_1 + 7x_2$ such that

$$x_1 + x_2 \le 3$$

$$x_1 + 2x_2 \le 7, \ x_1, \ x_2 \ge 0.$$

Then an optimal solution is given by

(A) (0, 3.5)

(B) (7,0)

(C) (3, 0)

(D) (0, 3).

69. Consider a balanced transportation problem with S sources of supply of an item and D destinations requiring the item. If the problem is to be formulated as an LPP the number of constraints in the LPP is

(A) Min (S, D)

(B) Max (S, D)

(C) SD

(D) S + D.

70. Consider a game played by 2 players with pay off matrix given by

	B_1	B_2	B_3	B_4
A_1	8	–2	9	–3
A_2	6	5	6	8
A_3	–2	4	–9	5

Then the game has

(A) no saddle point

(B) a saddle point which corresponds to the strategy (A_1, B_3)

(C) a saddle point which corresponds to the strategy (A_2, B_2)

(D) a saddle point which corresponds to the strategy (A_3, B_3).

71. If V_{rand}, V_{prop} and V_{opt} denote the variances of the estimated means under simple random sampling, proportional and optimum allocation in stratified random sampling respectively, then

(A) $V_{opt} \le V_{prop} \le V_{rand}$

(B) $V_{opt} \le V_{rand} \le V_{prop}$

(C) $V_{prop} \le V_{rand} \le V_{opt}$

(D) $V_{prop} \le V_{opt} \le V_{rand}$.

72. If EPC is ignored then V (P_{st}), variance of estimated proportion under stratified sampling is

(A) $\Sigma \frac{W_h^2 \; P_h \, Q_h}{N_h}$

(B) $\Sigma \frac{W_h \; P_h \, Q_h}{n_h}$

(C) $\Sigma \frac{W_h \; P_h \, Q_h}{N_h}$

(D) $\Sigma \frac{W_h^2 \; P_h \, Q_h}{N_h^2}$.

73. The following layout

A	B	C	D
B	A	D	C
C	D	A	B
A	B	A	D

corresponds to

(A) C R D

(B) R B D

(C) L S D

(D) B I B D.

74. If BC is confounded in a 2^3–factorial experiment, a block in a replicate will have

(A) (1), a, bc, abc

(B) a, bc, abc, b

(C) a, abc, c, b

(D) (1), b, bc, a.

ROUGH WORK