



# MATHEMATICAL SCIENCES

Question Booklet Sl. No.

010907

Name & Signature of the Invigilator

PAPER – II

OMR Answer Sheet No. :

CODE-01

Roll No. :

(in figures as in Hall Ticket)

Roll Number in words : .....

Time : 2 Hours]

No. of Printed Pages : 32

[Maximum Marks : 200

## Instructions for the Candidates

- Write your Roll Number in the space provided on the top of this page.
- This paper consists of **one hundred (100)** multiple choice type of questions. **All** questions are compulsory.
- At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as below :
  - To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker seal and do not accept an open booklet.
  - Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
  - After this verification is over, the Test Booklet Number should be entered on the OMR Answer Sheet and the OMR Answer Sheet Number should be entered on this Test Booklet.
- Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the oval as indicated below on the correct response against each item.  
**Example:** (A) (B) (C) (D) where (B) is the correct response.
- Your responses to the items are to be indicated on the OMR Answer Sheet under Paper – II only. If you mark your response at any place other than in the oval in the OMR Answer Sheet, it will not be evaluated.
- Rough Work is to be done in the end of this booklet.
- If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
- You have to return the original OMR Answer Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are however, allowed to carry original question booklet and duplicate copy of OMR Answer Sheet on conclusion of examination.
- Use only Blue/Black Ball point pen.
- Use of any calculator or any electronic devices or log table etc., are prohibited.
- There shall be no negative marking.

## પરીક્ષાર્થીઓ માટે સૂચનાઓ

- આ પાનાની ટોચ પર દર્શાવેલી જગ્યામાં તમારો રોલ નંબર લખો.
- આ પ્રશ્નપત્રમાં બહુવિકલ્પિક ઉત્તરો ધરાવતા સો (૧૦૦) પ્રશ્નો આપેલા છે. બધા જ પ્રશ્નો ફરજિયાત છે.
- પરીક્ષાની શરૂઆતમાં આપને પ્રશ્નપુસ્તિકા આપવામાં આવશે. પ્રથમ પાંચ (૫) મિનિટ દરમિયાન તમારે પ્રશ્નપુસ્તિકા ખોલી અને ફરજિયાતપણે નીચે મુજબ પરીક્ષણ કરવું :
  - પ્રશ્નપુસ્તિકાનો વપરાશ કરવા માટે આ કવર પૃષ્ઠની ધાર પર આપેલ સીલ સ્ટીકર ફાડી નાખો. કોઈપણ સંજોગોમાં સીલ સ્ટીકર વગરની કે ખુલ્લી પ્રશ્નપુસ્તિકા સ્વીકારશો નહીં.
  - કવર પૃષ્ઠ પર છપાયેલ નિર્દેશાનુસાર પ્રશ્નપુસ્તિકાના પ્રશ્નો, પૃષ્ઠો અને સંખ્યાને બરાબર ચકાસી લો. ખામીયુક્ત પ્રશ્નપુસ્તિકા કે જેમાં પ્રશ્નો/ પૃષ્ઠો ઓછાં હોય, બે વાર છપાયા હોય, અનુક્રમમાં અથવા અન્ય કોઈ ફરક હોય અર્થાત કોઈપણ સંજોગોમાં ખામીયુક્ત પ્રશ્નપુસ્તિકા સ્વીકારશો નહીં. અને જો ખામીયુક્ત પ્રશ્નપુસ્તિકા મળી હોય તો નિરીક્ષક પાસેથી તુરંત જ બીજી સારી પ્રશ્નપુસ્તિકા મેળવી લેવી. આ માટે ઉમેદવારને પાંચ (૫) મિનિટનો સમયગાળો આપવામાં આવશે. પછીથી, પ્રશ્નપુસ્તિકા બદલવામાં આવશે નહીં કે કોઈ વધારાનો સમયગાળો આપવામાં આવશે નહીં.
  - આ ચકાસણી સમાપ્ત થાય પછી, પ્રશ્નપુસ્તિકાનો નંબર OMR જવાબ પત્રક પર લખવો અને OMR જવાબ પત્રકનો નંબર પ્રશ્નપુસ્તિકા પર લખવો.
- પ્રત્યેક પ્રશ્ન માટે ચાર જવાબ વિકલ્પ (A), (B), (C) અને (D) આપવામાં આવેલ છે. તમારે સાચા જવાબના ઓવલ (oval) ને નીચે આપેલ ઉદાહરણ મુજબ પેનથી ભરીને સંપૂર્ણ કાળું કરવાનું રહેશે.  
**ઉદાહરણ :** (A) (B) (C) (D) કે જ્યાં (B) સાચો જવાબ છે.
- આ પ્રશ્નપુસ્તિકાના પ્રશ્નોના જવાબ અલગથી આપવામાં આવેલ OMR જવાબ પત્રકમાં પેપર-II લપેલ વિભાગમાં જ અંકિત કરવા. જો આપ OMR જવાબ પત્રકમાં આપેલ ઓવલ (oval) સિવાય અન્ય સ્થાને જવાબ અંકિત કરશો તો તે જવાબનું મૂલ્યાંકન કરવામાં આવશે નહીં.
- કાચું કામ (Rough work) પ્રશ્નપુસ્તિકાના અંતિમ પૃષ્ઠ પર કરવું.
- જો આપ OMR જવાબ પત્રક નિયત જગ્યા સિવાય અન્ય કોઈપણ સ્થાને, આપનું નામ, રોલ નંબર, ફોન નંબર અથવા એવું કોઈ ચિહ્ન જેનાથી તમારી ઓળખ થઈ શકે, અંકિત કરશો અથવા અલગ ભાષાનો પ્રયોગ કરો, અથવા અન્ય કોઈ અનુચિત સાધનોનો ઉપયોગ કરો, જેમકે અંકિત કરી દીધેલ જવાબ ભૂંસી નાખવો કે સફેદ શાહીનો ઉપયોગ કરી બદલશો તો આપને પરીક્ષા માટે અયોગ્ય જાહેર કરવામાં આવશે.
- પરીક્ષા સમય પૂરો થઈ ગયા બાદ ઓરીજનલ OMR જવાબ પત્રક જે તે નિરીક્ષકને ફરજિયાત સોપી દેવું અને કોઈ પણ સંજોગોમાં તે પરીક્ષા ખંડની બહાર લઈ જવું નહીં. પરીક્ષા પૂર્ણ થયા બાદ ઉમેદવાર ઓરીજનલ પ્રશ્નપુસ્તિકા અને OMR જવાબ પત્રકની ડુપ્લિકેટ કોપી પોતાની સાથે લઈ જઈ શકે છે.
- માત્ર કાળી / ભૂરી બોલ પોઇન્ટ પેન વાપરવી.
- કેલ્ક્યુલેટર, લોગ ટેબલ અને અન્ય ઇલેક્ટ્રોનિક યંત્રોનો ઉપયોગ કરવાની મનાઈ છે.
- ખોટા જવાબ માટે નકારાત્મક ગુણાંકન પ્રથા નથી.



010907

DO NOT WRITE HERE



# LOGARITHMS

|    | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | Mean Differences |   |    |    |    |    |    |    |    |
|----|------|------|------|------|------|------|------|------|------|------|------------------|---|----|----|----|----|----|----|----|
|    |      |      |      |      |      |      |      |      |      |      | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 10 | 0000 | 0043 | 0086 | 0128 | 0170 | 0212 | 0253 | 0294 | 0334 | 0374 | 4                | 8 | 12 | 17 | 21 | 25 | 29 | 33 | 37 |
| 11 | 0414 | 0453 | 0492 | 0531 | 0569 | 0607 | 0645 | 0682 | 0719 | 0755 | 4                | 8 | 11 | 15 | 19 | 23 | 28 | 30 | 34 |
| 12 | 0782 | 0828 | 0864 | 0899 | 0934 | 0969 | 1004 | 1038 | 1072 | 1106 | 3                | 7 | 10 | 14 | 17 | 21 | 24 | 28 | 31 |
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 | 1303 | 1335 | 1367 | 1399 | 1430 | 3                | 6 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 14 | 1481 | 1492 | 1523 | 1553 | 1584 | 1614 | 1644 | 1673 | 1703 | 1732 | 3                | 6 | 9  | 12 | 15 | 18 | 21 | 24 | 27 |
| 15 | 1761 | 1790 | 1818 | 1847 | 1875 | 1903 | 1931 | 1959 | 1987 | 2014 | 3                | 6 | 8  | 11 | 14 | 17 | 20 | 22 | 25 |
| 16 | 2041 | 2068 | 2095 | 2122 | 2148 | 2175 | 2201 | 2227 | 2253 | 2279 | 3                | 5 | 8  | 11 | 13 | 16 | 18 | 21 | 24 |
| 17 | 2304 | 2330 | 2355 | 2380 | 2405 | 2430 | 2455 | 2480 | 2504 | 2529 | 2                | 5 | 7  | 10 | 12 | 15 | 17 | 20 | 22 |
| 18 | 2553 | 2577 | 2601 | 2625 | 2648 | 2672 | 2695 | 2718 | 2742 | 2765 | 2                | 5 | 7  | 9  | 12 | 14 | 16 | 19 | 21 |
| 19 | 2788 | 2810 | 2833 | 2856 | 2878 | 2900 | 2923 | 2945 | 2967 | 2989 | 2                | 4 | 7  | 9  | 11 | 13 | 16 | 18 | 20 |
| 20 | 3010 | 3032 | 3054 | 3075 | 3096 | 3118 | 3139 | 3160 | 3181 | 3201 | 2                | 4 | 6  | 8  | 11 | 13 | 15 | 17 | 19 |
| 21 | 3222 | 3243 | 3263 | 3284 | 3304 | 3324 | 3345 | 3365 | 3385 | 3404 | 2                | 4 | 6  | 8  | 10 | 12 | 14 | 16 | 18 |
| 22 | 3424 | 3444 | 3464 | 3483 | 3502 | 3522 | 3541 | 3560 | 3579 | 3598 | 2                | 4 | 6  | 8  | 10 | 12 | 14 | 15 | 17 |
| 23 | 3617 | 3638 | 3655 | 3674 | 3692 | 3711 | 3729 | 3747 | 3766 | 3784 | 2                | 4 | 6  | 7  | 9  | 11 | 13 | 15 | 17 |
| 24 | 3802 | 3820 | 3838 | 3856 | 3874 | 3892 | 3909 | 3927 | 3945 | 3962 | 2                | 4 | 5  | 7  | 9  | 11 | 12 | 14 | 16 |
| 25 | 3979 | 3997 | 4014 | 4031 | 4048 | 4065 | 4082 | 4099 | 4116 | 4133 | 2                | 3 | 5  | 7  | 9  | 10 | 12 | 14 | 15 |
| 26 | 4150 | 4166 | 4183 | 4200 | 4216 | 4232 | 4249 | 4265 | 4281 | 4298 | 2                | 3 | 5  | 7  | 8  | 10 | 11 | 13 | 15 |
| 27 | 4314 | 4330 | 4346 | 4362 | 4378 | 4393 | 4409 | 4425 | 4440 | 4456 | 2                | 3 | 5  | 6  | 8  | 9  | 11 | 13 | 14 |
| 28 | 4472 | 4487 | 4502 | 4518 | 4533 | 4548 | 4564 | 4579 | 4594 | 4609 | 2                | 3 | 5  | 6  | 8  | 9  | 11 | 12 | 14 |
| 29 | 4624 | 4639 | 4654 | 4669 | 4683 | 4698 | 4713 | 4728 | 4742 | 4757 | 1                | 3 | 4  | 6  | 7  | 9  | 10 | 12 | 13 |
| 30 | 4771 | 4786 | 4800 | 4814 | 4829 | 4843 | 4857 | 4871 | 4886 | 4900 | 1                | 3 | 4  | 6  | 7  | 9  | 10 | 11 | 13 |
| 31 | 4914 | 4928 | 4942 | 4955 | 4969 | 4983 | 4997 | 5011 | 5024 | 5038 | 1                | 3 | 4  | 6  | 7  | 8  | 10 | 11 | 12 |
| 32 | 5051 | 5065 | 5079 | 5092 | 5106 | 5119 | 5132 | 5145 | 5159 | 5172 | 1                | 3 | 4  | 5  | 7  | 8  | 9  | 11 | 12 |
| 33 | 5185 | 5198 | 5211 | 5224 | 5237 | 5250 | 5263 | 5276 | 5289 | 5302 | 1                | 3 | 4  | 5  | 6  | 8  | 9  | 10 | 12 |
| 34 | 5315 | 5328 | 5340 | 5353 | 5366 | 5378 | 5391 | 5403 | 5416 | 5428 | 1                | 3 | 4  | 5  | 6  | 8  | 9  | 10 | 11 |
| 35 | 5441 | 5453 | 5465 | 5478 | 5490 | 5502 | 5514 | 5527 | 5539 | 5551 | 1                | 2 | 4  | 5  | 6  | 7  | 9  | 10 | 11 |
| 36 | 5563 | 5575 | 5587 | 5599 | 5611 | 5623 | 5636 | 5647 | 5658 | 5670 | 1                | 2 | 4  | 5  | 6  | 7  | 8  | 10 | 11 |
| 37 | 5682 | 5694 | 5705 | 5717 | 5729 | 5740 | 5752 | 5763 | 5775 | 5786 | 1                | 2 | 3  | 5  | 6  | 7  | 8  | 9  | 10 |
| 38 | 5798 | 5809 | 5821 | 5832 | 5843 | 5855 | 5866 | 5877 | 5888 | 5899 | 1                | 2 | 3  | 5  | 6  | 7  | 8  | 9  | 10 |
| 39 | 5911 | 5922 | 5933 | 5944 | 5955 | 5966 | 5977 | 5988 | 5999 | 6010 | 1                | 2 | 3  | 4  | 5  | 7  | 8  | 9  | 10 |
| 40 | 6021 | 6031 | 6042 | 6053 | 6064 | 6075 | 6085 | 6096 | 6107 | 6117 | 1                | 2 | 3  | 4  | 5  | 6  | 8  | 9  | 10 |
| 41 | 6128 | 6138 | 6149 | 6160 | 6170 | 6180 | 6191 | 6201 | 6212 | 6222 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 42 | 6232 | 6243 | 6253 | 6263 | 6274 | 6284 | 6294 | 6304 | 6314 | 6325 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 43 | 6335 | 6345 | 6355 | 6365 | 6375 | 6385 | 6395 | 6405 | 6415 | 6425 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 44 | 6435 | 6444 | 6454 | 6464 | 6474 | 6484 | 6493 | 6503 | 6513 | 6522 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 45 | 6532 | 6542 | 6551 | 6561 | 6571 | 6580 | 6590 | 6599 | 6609 | 6618 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 46 | 6628 | 6637 | 6646 | 6656 | 6665 | 6675 | 6684 | 6693 | 6702 | 6712 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 7  | 8  |
| 47 | 6721 | 6730 | 6739 | 6749 | 6758 | 6767 | 6776 | 6785 | 6794 | 6803 | 1                | 2 | 3  | 4  | 5  | 5  | 6  | 7  | 8  |
| 48 | 6812 | 6821 | 6830 | 6839 | 6848 | 6857 | 6866 | 6875 | 6884 | 6893 | 1                | 2 | 3  | 4  | 4  | 5  | 6  | 7  | 8  |
| 49 | 6902 | 6911 | 6920 | 6928 | 6937 | 6946 | 6955 | 6964 | 6972 | 6981 | 1                | 2 | 3  | 4  | 4  | 5  | 6  | 7  | 8  |
| 50 | 6990 | 6998 | 7007 | 7016 | 7024 | 7033 | 7042 | 7050 | 7059 | 7067 | 1                | 2 | 3  | 3  | 4  | 5  | 6  | 7  | 8  |
| 51 | 7076 | 7084 | 7093 | 7101 | 7110 | 7118 | 7126 | 7135 | 7143 | 7152 | 1                | 2 | 3  | 3  | 4  | 5  | 6  | 7  | 8  |
| 52 | 7160 | 7168 | 7177 | 7185 | 7193 | 7202 | 7210 | 7218 | 7226 | 7235 | 1                | 2 | 2  | 3  | 4  | 5  | 6  | 7  | 7  |
| 53 | 7243 | 7251 | 7259 | 7267 | 7275 | 7284 | 7292 | 7300 | 7308 | 7316 | 1                | 2 | 2  | 3  | 4  | 5  | 6  | 6  | 7  |
| 54 | 7324 | 7332 | 7340 | 7348 | 7356 | 7364 | 7372 | 7380 | 7388 | 7396 | 1                | 2 | 2  | 3  | 4  | 5  | 6  | 6  | 7  |

No.  $\log$   $\ln x = \log_e x = (1/M) \log_{10} x$   $(1/M) = 2.30259$   $\log$   
 $x = 3.14159$   $0.49715$   $\log x = \log_{10} x = M \log_e x$   $M = 0.43429$   $\bar{1} 63778$   
 $e = 2.71828$   $0.43429$



# LOGARITHMS

|    | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | Mean Differences |   |   |   |   |   |   |   |   |
|----|------|------|------|------|------|------|------|------|------|------|------------------|---|---|---|---|---|---|---|---|
|    |      |      |      |      |      |      |      |      |      |      | 1                | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 55 | 7404 | 7412 | 7419 | 7427 | 7435 | 7443 | 7451 | 7459 | 7466 | 7474 | 1                | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 56 | 7482 | 7490 | 7497 | 7505 | 7513 | 7520 | 7528 | 7536 | 7543 | 7551 | 1                | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 57 | 7559 | 7566 | 7574 | 7582 | 7589 | 7597 | 7604 | 7612 | 7619 | 7627 | 1                | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 58 | 7634 | 7642 | 7649 | 7657 | 7664 | 7672 | 7679 | 7686 | 7694 | 7701 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |
| 59 | 7709 | 7716 | 7723 | 7731 | 7738 | 7745 | 7752 | 7760 | 7767 | 7774 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |
| 60 | 7782 | 7789 | 7796 | 7803 | 7810 | 7818 | 7825 | 7832 | 7839 | 7846 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 8 |
| 61 | 7853 | 7860 | 7868 | 7875 | 7882 | 7889 | 7896 | 7903 | 7910 | 7917 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 8 |
| 62 | 7924 | 7931 | 7938 | 7945 | 7952 | 7959 | 7966 | 7973 | 7980 | 7987 | 1                | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 8 |
| 63 | 7993 | 8000 | 8007 | 8014 | 8021 | 8028 | 8035 | 8041 | 8048 | 8055 | 1                | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 8 |
| 64 | 8062 | 8069 | 8075 | 8082 | 8089 | 8096 | 8102 | 8109 | 8116 | 8122 | 1                | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 8 |
| 65 | 8129 | 8136 | 8142 | 8149 | 8156 | 8162 | 8169 | 8176 | 8182 | 8189 | 1                | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 8 |
| 66 | 8195 | 8202 | 8209 | 8215 | 8222 | 8228 | 8235 | 8241 | 8248 | 8254 | 1                | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 67 | 8261 | 8267 | 8274 | 8280 | 8287 | 8293 | 8299 | 8306 | 8312 | 8319 | 1                | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 68 | 8325 | 8331 | 8338 | 8344 | 8351 | 8357 | 8363 | 8370 | 8376 | 8382 | 1                | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 |
| 69 | 8388 | 8395 | 8401 | 8407 | 8414 | 8420 | 8426 | 8432 | 8439 | 8445 | 1                | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| 70 | 8451 | 8457 | 8463 | 8470 | 8476 | 8482 | 8488 | 8494 | 8500 | 8506 | 1                | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| 71 | 8513 | 8519 | 8525 | 8531 | 8537 | 8543 | 8549 | 8555 | 8561 | 8567 | 1                | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| 72 | 8573 | 8579 | 8585 | 8591 | 8597 | 8603 | 8609 | 8615 | 8621 | 8627 | 1                | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| 73 | 8633 | 8639 | 8645 | 8651 | 8657 | 8663 | 8669 | 8675 | 8681 | 8686 | 1                | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| 74 | 8692 | 8698 | 8704 | 8710 | 8716 | 8722 | 8727 | 8733 | 8739 | 8745 | 1                | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| 75 | 8751 | 8756 | 8762 | 8768 | 8774 | 8779 | 8785 | 8791 | 8797 | 8802 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 6 |
| 76 | 8808 | 8814 | 8820 | 8825 | 8831 | 8837 | 8842 | 8848 | 8854 | 8859 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 6 |
| 77 | 8865 | 8871 | 8876 | 8882 | 8887 | 8893 | 8899 | 8904 | 8910 | 8915 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 78 | 8921 | 8927 | 8932 | 8938 | 8943 | 8949 | 8954 | 8960 | 8965 | 8971 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 79 | 8976 | 8982 | 8987 | 8993 | 8998 | 9004 | 9009 | 9015 | 9020 | 9025 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 80 | 9031 | 9036 | 9042 | 9047 | 9053 | 9058 | 9063 | 9069 | 9074 | 9079 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 81 | 9085 | 9090 | 9096 | 9101 | 9106 | 9112 | 9117 | 9122 | 9128 | 9133 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 82 | 9138 | 9143 | 9149 | 9154 | 9159 | 9165 | 9170 | 9175 | 9180 | 9186 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 83 | 9191 | 9196 | 9201 | 9206 | 9212 | 9217 | 9222 | 9227 | 9232 | 9238 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 84 | 9243 | 9248 | 9253 | 9258 | 9263 | 9269 | 9274 | 9279 | 9284 | 9289 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 85 | 9294 | 9299 | 9304 | 9309 | 9315 | 9320 | 9325 | 9330 | 9335 | 9340 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 86 | 9346 | 9350 | 9355 | 9360 | 9365 | 9370 | 9375 | 9380 | 9385 | 9390 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 87 | 9395 | 9400 | 9405 | 9410 | 9415 | 9420 | 9425 | 9430 | 9435 | 9440 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 88 | 9445 | 9450 | 9455 | 9460 | 9465 | 9469 | 9474 | 9479 | 9484 | 9489 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 89 | 9494 | 9499 | 9504 | 9509 | 9513 | 9518 | 9523 | 9528 | 9533 | 9538 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 90 | 9542 | 9547 | 9552 | 9557 | 9562 | 9566 | 9571 | 9576 | 9581 | 9586 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 91 | 9590 | 9595 | 9600 | 9605 | 9609 | 9614 | 9619 | 9624 | 9628 | 9633 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 92 | 9638 | 9643 | 9647 | 9652 | 9657 | 9661 | 9666 | 9671 | 9675 | 9680 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 93 | 9685 | 9689 | 9694 | 9699 | 9703 | 9708 | 9713 | 9717 | 9722 | 9727 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 94 | 9731 | 9736 | 9741 | 9745 | 9750 | 9754 | 9759 | 9763 | 9768 | 9773 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 95 | 9777 | 9782 | 9786 | 9791 | 9795 | 9800 | 9805 | 9809 | 9814 | 9818 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 96 | 9823 | 9827 | 9832 | 9836 | 9841 | 9845 | 9850 | 9854 | 9859 | 9863 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 97 | 9868 | 9872 | 9877 | 9881 | 9886 | 9890 | 9894 | 9899 | 9903 | 9908 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 98 | 9912 | 9917 | 9921 | 9926 | 9930 | 9934 | 9939 | 9943 | 9948 | 9952 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 99 | 9956 | 9961 | 9965 | 9969 | 9974 | 9978 | 9983 | 9987 | 9991 | 9995 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |

| P                  | 1      | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 10     |
|--------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| log e <sup>+</sup> | 0.4343 | 0.8686 | 1.3029 | 1.7372 | 2.1715 | 2.6058 | 3.0401 | 3.4744 | 3.9087 | 4.3429 |
| log e <sup>-</sup> | 1.5657 | 1.1314 | 2.6971 | 2.2628 | 3.8285 | 3.3942 | 4.9599 | 4.5256 | 4.0913 | 5.6571 |



# ANTILOGARITHMS

|     | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | Mean Differences |   |   |   |   |   |   |   |   |
|-----|------|------|------|------|------|------|------|------|------|------|------------------|---|---|---|---|---|---|---|---|
|     |      |      |      |      |      |      |      |      |      |      | 1                | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| .00 | 1000 | 1002 | 1005 | 1007 | 1009 | 1012 | 1014 | 1016 | 1019 | 1021 | 0                | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| .01 | 1023 | 1026 | 1028 | 1030 | 1033 | 1035 | 1038 | 1040 | 1042 | 1045 | 0                | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| .02 | 1047 | 1050 | 1052 | 1054 | 1057 | 1059 | 1062 | 1064 | 1067 | 1069 | 0                | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| .03 | 1072 | 1074 | 1076 | 1079 | 1081 | 1084 | 1086 | 1089 | 1091 | 1094 | 0                | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| .04 | 1096 | 1099 | 1102 | 1104 | 1107 | 1109 | 1112 | 1114 | 1117 | 1119 | 0                | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| .05 | 1122 | 1125 | 1127 | 1130 | 1132 | 1135 | 1138 | 1140 | 1143 | 1146 | 0                | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| .06 | 1148 | 1151 | 1153 | 1156 | 1159 | 1161 | 1164 | 1167 | 1169 | 1172 | 0                | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| .07 | 1175 | 1178 | 1180 | 1183 | 1186 | 1189 | 1191 | 1194 | 1197 | 1199 | 0                | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| .08 | 1202 | 1205 | 1208 | 1211 | 1213 | 1216 | 1219 | 1222 | 1225 | 1227 | 0                | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 |
| .09 | 1230 | 1233 | 1236 | 1239 | 1242 | 1245 | 1247 | 1250 | 1253 | 1256 | 0                | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 |
| .10 | 1259 | 1262 | 1265 | 1268 | 1271 | 1274 | 1276 | 1279 | 1282 | 1285 | 0                | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 |
| .11 | 1288 | 1291 | 1294 | 1297 | 1300 | 1303 | 1306 | 1309 | 1312 | 1315 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .12 | 1318 | 1321 | 1324 | 1327 | 1330 | 1334 | 1337 | 1340 | 1343 | 1346 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .13 | 1349 | 1352 | 1355 | 1358 | 1361 | 1365 | 1368 | 1371 | 1374 | 1377 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .14 | 1380 | 1384 | 1387 | 1390 | 1393 | 1396 | 1400 | 1403 | 1406 | 1409 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .15 | 1413 | 1416 | 1419 | 1422 | 1426 | 1429 | 1432 | 1435 | 1439 | 1442 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .16 | 1445 | 1449 | 1452 | 1455 | 1459 | 1462 | 1466 | 1469 | 1472 | 1476 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .17 | 1479 | 1483 | 1486 | 1489 | 1493 | 1496 | 1500 | 1503 | 1507 | 1510 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .18 | 1514 | 1517 | 1521 | 1524 | 1528 | 1531 | 1535 | 1538 | 1542 | 1545 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .19 | 1549 | 1552 | 1556 | 1560 | 1563 | 1567 | 1570 | 1574 | 1578 | 1581 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .20 | 1585 | 1589 | 1592 | 1596 | 1600 | 1603 | 1607 | 1611 | 1614 | 1618 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .21 | 1622 | 1626 | 1629 | 1633 | 1637 | 1641 | 1644 | 1648 | 1652 | 1656 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .22 | 1660 | 1663 | 1667 | 1671 | 1675 | 1679 | 1683 | 1687 | 1690 | 1694 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .23 | 1698 | 1702 | 1706 | 1710 | 1714 | 1718 | 1722 | 1726 | 1730 | 1734 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .24 | 1738 | 1742 | 1746 | 1750 | 1754 | 1758 | 1762 | 1766 | 1770 | 1774 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .25 | 1778 | 1782 | 1786 | 1791 | 1795 | 1799 | 1803 | 1807 | 1811 | 1816 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .26 | 1820 | 1824 | 1828 | 1832 | 1837 | 1841 | 1845 | 1849 | 1854 | 1858 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .27 | 1862 | 1866 | 1871 | 1875 | 1879 | 1884 | 1888 | 1892 | 1897 | 1901 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .28 | 1905 | 1910 | 1914 | 1919 | 1923 | 1928 | 1932 | 1936 | 1941 | 1946 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .29 | 1950 | 1954 | 1959 | 1963 | 1968 | 1972 | 1977 | 1982 | 1986 | 1991 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .30 | 1995 | 2000 | 2004 | 2009 | 2014 | 2018 | 2023 | 2028 | 2032 | 2037 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .31 | 2042 | 2046 | 2051 | 2056 | 2061 | 2065 | 2070 | 2075 | 2080 | 2084 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .32 | 2089 | 2094 | 2099 | 2104 | 2109 | 2113 | 2118 | 2123 | 2128 | 2133 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .33 | 2138 | 2143 | 2148 | 2153 | 2158 | 2163 | 2168 | 2173 | 2178 | 2183 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| .34 | 2188 | 2193 | 2198 | 2203 | 2208 | 2213 | 2218 | 2223 | 2228 | 2234 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |
| .35 | 2239 | 2244 | 2249 | 2254 | 2259 | 2264 | 2270 | 2275 | 2280 | 2286 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |
| .36 | 2291 | 2296 | 2301 | 2307 | 2312 | 2317 | 2323 | 2328 | 2333 | 2339 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |
| .37 | 2344 | 2350 | 2355 | 2360 | 2366 | 2371 | 2377 | 2382 | 2388 | 2393 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |
| .38 | 2399 | 2404 | 2410 | 2415 | 2421 | 2427 | 2432 | 2438 | 2443 | 2449 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |
| .39 | 2455 | 2460 | 2466 | 2472 | 2477 | 2483 | 2489 | 2495 | 2500 | 2506 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |
| .40 | 2512 | 2518 | 2523 | 2529 | 2535 | 2541 | 2547 | 2553 | 2559 | 2564 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |
| .41 | 2570 | 2576 | 2582 | 2588 | 2594 | 2600 | 2606 | 2612 | 2618 | 2624 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |
| .42 | 2630 | 2636 | 2642 | 2649 | 2655 | 2661 | 2667 | 2673 | 2679 | 2685 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |
| .43 | 2692 | 2698 | 2704 | 2710 | 2716 | 2723 | 2729 | 2735 | 2742 | 2748 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |
| .44 | 2754 | 2761 | 2767 | 2773 | 2780 | 2786 | 2793 | 2799 | 2805 | 2812 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |
| .45 | 2818 | 2825 | 2831 | 2838 | 2844 | 2851 | 2858 | 2864 | 2871 | 2877 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |
| .46 | 2884 | 2891 | 2897 | 2904 | 2911 | 2917 | 2924 | 2931 | 2938 | 2944 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |
| .47 | 2951 | 2958 | 2965 | 2972 | 2979 | 2985 | 2992 | 2999 | 3006 | 3013 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |
| .48 | 3020 | 3027 | 3034 | 3041 | 3048 | 3055 | 3062 | 3069 | 3076 | 3083 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |
| .49 | 3090 | 3097 | 3105 | 3112 | 3119 | 3126 | 3133 | 3141 | 3148 | 3155 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |



# ANTILOGARITHMS

|     | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | Mean Differences |   |   |   |    |    |    |    |    |
|-----|------|------|------|------|------|------|------|------|------|------|------------------|---|---|---|----|----|----|----|----|
|     |      |      |      |      |      |      |      |      |      |      | 1                | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  |
| .50 | 3182 | 3170 | 3177 | 3184 | 3192 | 3199 | 3206 | 3214 | 3221 | 3228 | 1                | 1 | 2 | 3 | 4  | 4  | 5  | 6  | 7  |
| .51 | 3236 | 3243 | 3251 | 3259 | 3266 | 3273 | 3281 | 3289 | 3296 | 3304 | 1                | 2 | 2 | 3 | 4  | 5  | 5  | 6  | 7  |
| .52 | 3311 | 3319 | 3327 | 3334 | 3342 | 3350 | 3357 | 3365 | 3373 | 3381 | 1                | 2 | 2 | 3 | 4  | 5  | 5  | 6  | 7  |
| .53 | 3388 | 3396 | 3404 | 3412 | 3420 | 3428 | 3436 | 3443 | 3451 | 3459 | 1                | 2 | 2 | 3 | 4  | 5  | 6  | 6  | 7  |
| .54 | 3467 | 3475 | 3483 | 3491 | 3499 | 3506 | 3516 | 3524 | 3532 | 3540 | 1                | 2 | 2 | 3 | 4  | 5  | 6  | 6  | 7  |
| .55 | 3548 | 3556 | 3565 | 3573 | 3581 | 3589 | 3597 | 3606 | 3614 | 3622 | 1                | 2 | 2 | 3 | 4  | 5  | 6  | 7  | 7  |
| .56 | 3631 | 3639 | 3648 | 3656 | 3664 | 3673 | 3681 | 3690 | 3698 | 3707 | 1                | 2 | 3 | 3 | 4  | 5  | 6  | 7  | 8  |
| .57 | 3715 | 3724 | 3733 | 3741 | 3750 | 3758 | 3767 | 3776 | 3784 | 3793 | 1                | 2 | 3 | 3 | 4  | 5  | 6  | 7  | 8  |
| .58 | 3802 | 3811 | 3819 | 3828 | 3837 | 3846 | 3855 | 3864 | 3873 | 3882 | 1                | 2 | 3 | 4 | 4  | 5  | 6  | 7  | 8  |
| .59 | 3890 | 3899 | 3908 | 3917 | 3926 | 3936 | 3945 | 3954 | 3963 | 3972 | 1                | 2 | 3 | 4 | 5  | 5  | 6  | 7  | 8  |
| .60 | 3981 | 3990 | 3999 | 4009 | 4018 | 4027 | 4036 | 4046 | 4055 | 4064 | 1                | 2 | 3 | 4 | 5  | 6  | 6  | 7  | 8  |
| .61 | 4074 | 4083 | 4093 | 4102 | 4111 | 4121 | 4130 | 4140 | 4150 | 4159 | 1                | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  |
| .62 | 4169 | 4178 | 4188 | 4198 | 4207 | 4217 | 4227 | 4236 | 4246 | 4256 | 1                | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  |
| .63 | 4266 | 4276 | 4285 | 4295 | 4305 | 4315 | 4325 | 4335 | 4345 | 4355 | 1                | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  |
| .64 | 4365 | 4375 | 4385 | 4395 | 4406 | 4416 | 4426 | 4436 | 4446 | 4457 | 1                | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  |
| .65 | 4467 | 4477 | 4487 | 4498 | 4508 | 4519 | 4529 | 4539 | 4550 | 4560 | 1                | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  |
| .66 | 4571 | 4581 | 4592 | 4603 | 4613 | 4624 | 4634 | 4645 | 4656 | 4667 | 1                | 2 | 3 | 4 | 5  | 6  | 7  | 9  | 10 |
| .67 | 4677 | 4688 | 4699 | 4710 | 4721 | 4732 | 4742 | 4753 | 4764 | 4775 | 1                | 2 | 3 | 4 | 5  | 7  | 8  | 9  | 10 |
| .68 | 4786 | 4797 | 4808 | 4819 | 4831 | 4842 | 4853 | 4864 | 4875 | 4887 | 1                | 2 | 3 | 4 | 6  | 7  | 8  | 9  | 10 |
| .69 | 4898 | 4909 | 4920 | 4932 | 4943 | 4956 | 4966 | 4977 | 4989 | 5000 | 1                | 2 | 3 | 5 | 6  | 7  | 8  | 9  | 10 |
| .70 | 5012 | 5023 | 5035 | 5047 | 5058 | 5070 | 5082 | 5093 | 5105 | 5117 | 1                | 2 | 4 | 5 | 6  | 7  | 8  | 9  | 11 |
| .71 | 5129 | 5140 | 5152 | 5164 | 5176 | 5188 | 5200 | 5212 | 5224 | 5236 | 1                | 2 | 4 | 5 | 6  | 7  | 8  | 10 | 11 |
| .72 | 5248 | 5260 | 5272 | 5284 | 5297 | 5309 | 5321 | 5333 | 5346 | 5358 | 1                | 2 | 4 | 5 | 6  | 7  | 9  | 10 | 11 |
| .73 | 5370 | 5383 | 5396 | 5408 | 5420 | 5433 | 5446 | 5458 | 5470 | 5483 | 1                | 3 | 4 | 5 | 6  | 8  | 9  | 10 | 11 |
| .74 | 5496 | 5508 | 5521 | 5534 | 5546 | 5559 | 5572 | 5585 | 5598 | 5610 | 1                | 3 | 4 | 5 | 6  | 8  | 9  | 10 | 12 |
| .75 | 5623 | 5636 | 5649 | 5662 | 5675 | 5689 | 5702 | 5715 | 5728 | 5741 | 1                | 3 | 4 | 5 | 7  | 8  | 9  | 10 | 12 |
| .76 | 5754 | 5768 | 5781 | 5794 | 5808 | 5821 | 5834 | 5848 | 5861 | 5875 | 1                | 3 | 4 | 5 | 7  | 8  | 9  | 11 | 12 |
| .77 | 5888 | 5902 | 5916 | 5929 | 5943 | 5957 | 5970 | 5984 | 5998 | 6012 | 1                | 3 | 4 | 6 | 7  | 8  | 10 | 11 | 12 |
| .78 | 6026 | 6039 | 6053 | 6067 | 6081 | 6096 | 6109 | 6124 | 6138 | 6152 | 1                | 3 | 4 | 6 | 7  | 8  | 10 | 11 | 13 |
| .79 | 6166 | 6180 | 6194 | 6209 | 6223 | 6237 | 6252 | 6266 | 6281 | 6295 | 1                | 3 | 4 | 6 | 7  | 9  | 10 | 11 | 13 |
| .80 | 6310 | 6324 | 6339 | 6353 | 6368 | 6383 | 6397 | 6412 | 6427 | 6442 | 1                | 3 | 4 | 6 | 7  | 9  | 10 | 12 | 13 |
| .81 | 6457 | 6471 | 6486 | 6501 | 6516 | 6531 | 6546 | 6561 | 6577 | 6592 | 2                | 3 | 5 | 6 | 8  | 9  | 11 | 12 | 14 |
| .82 | 6607 | 6622 | 6637 | 6653 | 6668 | 6683 | 6699 | 6714 | 6730 | 6745 | 2                | 3 | 5 | 6 | 8  | 9  | 11 | 12 | 14 |
| .83 | 6761 | 6776 | 6792 | 6808 | 6823 | 6839 | 6855 | 6871 | 6887 | 6902 | 2                | 3 | 5 | 6 | 8  | 9  | 11 | 13 | 14 |
| .84 | 6918 | 6934 | 6950 | 6966 | 6982 | 6998 | 7015 | 7031 | 7047 | 7063 | 2                | 3 | 5 | 6 | 8  | 10 | 11 | 13 | 15 |
| .85 | 7079 | 7096 | 7112 | 7129 | 7145 | 7161 | 7178 | 7194 | 7211 | 7228 | 2                | 3 | 5 | 7 | 8  | 10 | 12 | 13 | 15 |
| .86 | 7244 | 7261 | 7278 | 7295 | 7311 | 7328 | 7345 | 7362 | 7379 | 7396 | 2                | 3 | 6 | 7 | 8  | 10 | 12 | 13 | 15 |
| .87 | 7413 | 7430 | 7447 | 7464 | 7482 | 7499 | 7516 | 7534 | 7551 | 7568 | 2                | 3 | 6 | 7 | 9  | 10 | 12 | 14 | 16 |
| .88 | 7586 | 7603 | 7621 | 7638 | 7656 | 7674 | 7691 | 7709 | 7727 | 7745 | 2                | 4 | 6 | 7 | 9  | 11 | 12 | 14 | 16 |
| .89 | 7762 | 7780 | 7798 | 7816 | 7834 | 7852 | 7870 | 7889 | 7907 | 7925 | 2                | 4 | 6 | 7 | 9  | 11 | 13 | 14 | 16 |
| .90 | 7943 | 7962 | 7980 | 7998 | 8017 | 8035 | 8054 | 8072 | 8091 | 8110 | 2                | 4 | 6 | 7 | 9  | 11 | 13 | 15 | 17 |
| .91 | 8128 | 8147 | 8166 | 8185 | 8204 | 8222 | 8241 | 8260 | 8279 | 8299 | 2                | 4 | 6 | 8 | 9  | 11 | 13 | 15 | 17 |
| .92 | 8318 | 8337 | 8356 | 8375 | 8395 | 8414 | 8433 | 8453 | 8472 | 8492 | 2                | 4 | 6 | 8 | 10 | 12 | 14 | 15 | 17 |
| .93 | 8511 | 8531 | 8551 | 8570 | 8590 | 8610 | 8630 | 8650 | 8670 | 8690 | 2                | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| .94 | 8710 | 8730 | 8750 | 8770 | 8790 | 8810 | 8831 | 8851 | 8872 | 8892 | 2                | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| .95 | 8913 | 8933 | 8954 | 8974 | 8995 | 9016 | 9036 | 9057 | 9078 | 9099 | 2                | 4 | 6 | 8 | 10 | 12 | 15 | 17 | 19 |
| .96 | 9120 | 9141 | 9162 | 9183 | 9204 | 9226 | 9247 | 9268 | 9290 | 9311 | 2                | 4 | 6 | 8 | 11 | 13 | 15 | 17 | 19 |
| .97 | 9333 | 9354 | 9376 | 9397 | 9418 | 9441 | 9462 | 9484 | 9506 | 9528 | 2                | 4 | 7 | 9 | 11 | 13 | 16 | 17 | 20 |
| .98 | 9550 | 9572 | 9594 | 9616 | 9638 | 9661 | 9683 | 9705 | 9727 | 9750 | 2                | 4 | 7 | 9 | 11 | 13 | 16 | 18 | 20 |
| .99 | 9772 | 9795 | 9817 | 9840 | 9863 | 9886 | 9908 | 9931 | 9954 | 9977 | 2                | 5 | 7 | 9 | 11 | 14 | 16 | 18 | 20 |



## MATHEMATICAL SCIENCES

### Paper – II

1. Let  $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ . Then which of the following is true ?

- (A)  $f$  is continuous at  $x = 0$  but not differentiable at 0
- (B)  $f$  is differentiable at  $x = 0$  and  $f'(0) = 0$
- (C)  $f$  is differentiable at  $x = 0$  and  $f'(0) = 1$
- (D)  $f$  is not continuous at  $x = 0$

2. Let  $S_1$  be the set of roots of all the polynomial equations in one variable with integer coefficients and  $S_2$  be the set of all functions  $f : \mathbb{N} \rightarrow \{0, 1, 2\}$  where  $\mathbb{N}$  is the set of all natural numbers. Then,

- (A) Both  $S_1$  and  $S_2$  are countable
- (B)  $S_1$  is countable and  $S_2$  is uncountable
- (C)  $S_1$  is uncountable and  $S_2$  is countable
- (D) Both  $S_1$  and  $S_2$  are uncountable

3. Let  $a_n = 1 + (-1)^n + \frac{(-1)^n}{n}$  for all  $n \in \mathbb{N}$ .

Then  $\limsup_{n \rightarrow \infty} \{a_n\} = \underline{\hspace{2cm}}$  and  $\liminf_{n \rightarrow \infty} \{a_n\} = \underline{\hspace{2cm}}$

- (A) 2 and 0
- (B) 0 and -2
- (C) 0 and 0
- (D) 2 and -2



4. The improper integral  $\int_3^{\infty} \frac{dx}{x^2 + x - 2}$  \_\_\_\_\_.

(A) is divergent

(B) is convergent and its value is  $\frac{1}{3} \ln\left(\frac{5}{2}\right)$

(C) is convergent and its value is  $\frac{1}{3} \ln\left(\frac{2}{5}\right)$

(D) is convergent but exact value cannot be evaluated

5. Let  $g(x) = \begin{cases} 0 & \text{if } 0 \leq x \leq \frac{1}{2} \\ 1 & \text{if } \frac{1}{2} < x \leq 1 \end{cases}$  and  $f_n(x) = \begin{cases} g(x) & \text{if } n \text{ is even} \\ g(1-x) & \text{if } n \text{ is odd} \end{cases}$ .

Then  $\liminf_{n \rightarrow \infty} f_n(x) = \underline{\hspace{2cm}}$  and  $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx = \underline{\hspace{2cm}}$

(A) 1 and 0

(B)  $\frac{1}{2}$  and 0

(C) 0 and 1

(D) 0 and  $\frac{1}{2}$

6. Let  $f(x, y) = ye^x$ . Then the rate of change of  $f$  at the point  $P(0, 2)$  in the direction from  $P$  to  $Q\left(2, \frac{1}{2}\right)$  is

(A) 2.5

(B) 1

(C)  $\sqrt{5}$

(D) 2

7. Let  $d$  be the discrete metric defined on  $\mathbb{R}$ , that is, for all  $x, y \in \mathbb{R}$ ,

$d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}$ . Then the family of all compact subsets on  $(\mathbb{R}, d)$  is

(A) The family of all the singleton sets

(B) The power set of  $\mathbb{R}$

(C) The family of all closed bounded intervals in  $\mathbb{R}$

(D) The family of all finite subsets of  $\mathbb{R}$



8. Consider  $\mathbb{R}$  with usual topology. Let  $O_n = \left(-\frac{1}{n}, 1 + \frac{1}{n}\right]$  for  $n = 1, 2, \dots$  and  $O = \bigcap_{n=1}^{\infty} O_n$ .

Then which one of the following is true ?

- (A) Each of  $O_n$ 's and  $O$  are open sets
  - (B) Each of  $O_n$ 's is not open, but  $O$  is open
  - (C) Each of  $O_n$ 's and  $O$  are closed sets
  - (D) Each of  $O_n$ 's is neither open nor closed and  $O$  is closed
9. Let  $(Y, \mathcal{F})$  be a subspace of a topological space  $(X, \tau)$ . Then which of the following is true ?
- (A) If  $A$  is open in  $(Y, \mathcal{F})$  then  $A$  is open in  $(X, \tau)$
  - (B) If  $A$  is closed in  $(Y, \mathcal{F})$  then  $A$  is closed in  $(X, \tau)$
  - (C) If  $A$  is closed in  $Y$ , then there is a  $\tau$  closed set  $H$  in  $X$  such that  $A = H \cap Y$
  - (D) If  $A$  is closed in  $(X, \tau)$  then  $A$  is closed in  $(Y, \mathcal{F})$
10. Let  $X$  be an infinite set and let  $p, q \in X$  with  $p \neq q$ . Let a subset  $G$  of  $X$  be open if and only if  $G = X$  or  $G \cap \{p, q\} = \emptyset$ .  
Let  $S = X \setminus \{p\}$  and  $T = X \setminus \{q\}$  and  $P = S \cap T$ . Then which of the following is false ?
- (A)  $S$  is compact in  $X$
  - (B)  $T$  is compact in  $X$
  - (C)  $X$  itself is compact
  - (D)  $P$  is compact

11. Let  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ . Then  $\lambda = 0$  is an eigenvalue of  $A$ . What is the dimension of the eigenspace with respect to the eigenvalue  $\lambda$  ?

- (A) 1
- (B) 2
- (C) 3
- (D) 0



12. Let  $A$  be a  $3 \times 3$  real matrix whose eigenvalues are  $1, -1$  and  $2$ . Then which of the following is NOT true ?
- (A)  $A^3 - 2A^2 - A + 2I = 0$
- (B)  $A^4 = 3A^3 + A^2 - 2A$
- (C)  $(A^2 + I)(A - 2I) = 0$
- (D)  $A^{-1} = \frac{-A^2 + 2A + I}{2}$
13. Let  $M(2, \mathbb{R})$  denote the set of all  $2 \times 2$  real matrices. Let  $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ . Define  $U = \{X \in M(2, \mathbb{R}) : AX = X\}$ . Then
- (A)  $\dim(U) = 1$
- (B)  $\dim(U) = 2$
- (C)  $\dim(U) = 3$
- (D)  $U$  is not a subspace of  $M(2, \mathbb{R})$
14. Let  $\mathbb{P}_n(\mathbb{R})$  represent the set of all polynomial of degree less than or equal to  $n$  over  $\mathbb{R}$ . Let  $S, T : \mathbb{P}_n(\mathbb{R}) \rightarrow \mathbb{P}_n(\mathbb{R})$  be linear transformations defined as  $T(p(x)) = p(x) + x p'(x)$  and  $S(p(x)) = p(x) - x p'(x)$ . Then
- (A) Both  $S$  and  $T$  are onto
- (B) Both  $S$  and  $T$  are one-one
- (C)  $T$  is one-one but  $S$  is not one-one
- (D)  $T$  is not one-one but  $S$  is one-one
15. What is the diagonal quadratic form of  $2x_1^2 + 4x_1x_2 + 5x_2^2$  ?
- (A)  $2y_1^2 + 4y_2^2 = 2x_1^2 + 4x_2^2$
- (B)  $6y_1^2 + y_2^2 = 6x_1^2 + x_2^2$
- (C)  $10y_1^2 - 4y_2^2 = 10x_1^2 - 4x_2^2$
- (D)  $y_1^2 - 6y_2^2 = y_1^2 - 6x_2^2$



16. Let  $A$  be a  $5 \times 5$  real matrix whose Jordan canonical form is given by

$$\begin{bmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

Then what is the minimum polynomial of  $A$  ?

- (A)  $(x - 4)(x - 2)$  (B)  $(x - 4)^2(x - 2)^2$   
(C)  $(x - 4)^2(x - 2)$  (D)  $(x - 4)^3(x - 2)^2$
17. Let  $T : V \rightarrow W$  be a linear transformation from a vector space  $V$  to a vector space  $W$ . Then which one of the following is correct ?
- (A) If  $\{v_1, v_2, \dots, v_k\}$  is linearly independent then  $\{T(v_1), T(v_2), \dots, T(v_k)\}$  is linearly independent  
(B) If  $\{T(v_1), T(v_2), \dots, T(v_k)\}$  is linearly independent then  $\{v_1, v_2, \dots, v_k\}$  is linearly independent  
(C) If  $\{v_1, v_2, \dots, v_k\}$  spans  $V$  then  $\{T(v_1), T(v_2), \dots, T(v_k)\}$  spans  $W$   
(D) If  $\{T(v_1), T(v_2), \dots, T(v_k)\}$  spans  $W$  then  $\{v_1, v_2, \dots, v_k\}$  spans  $V$
18. Fix a  $3 \times 3$  real matrix of rank 1. Let  $M(3, \mathbb{R})$  denote the set of all  $3 \times 3$  real matrices. Define a linear transformation  $T : M(3, \mathbb{R}) \rightarrow M(3, \mathbb{R})$  given by  $T(X) = AX$ . Then what is dimension of  $\text{Ker}(T)$  ?
- (A) 3 (B) 0  
(C) 6 (D) 4
19. Suppose  $f(z)$  is a bilinear transformation that maps  $-1$  to  $0$ ,  $1$  to  $\infty$  and  $0$  to  $-1$ . Then what is  $f(\infty)$  ?
- (A) 1 (B) 2  
(C)  $-2$  (D)  $\frac{1}{2}$



20. Let  $\mathbb{C}^*$  denote the set of non-zero complex numbers. Define  $f : \mathbb{C}^* \rightarrow \mathbb{C}$  as  $f(z) = ze^{1/z}$ . Then which of the following is true ?
- (A) 0 is an essential singularity of  $f$
  - (B) 0 is a removable singularity of  $f$
  - (C) 0 is a simple pole of  $f$
  - (D)  $\text{Res}(f; 0) = 0$
21. Let  $z = x + iy$ . Define  $f(z) = \cos(x) - i\sin(x)$  and  $g(z) = \cos(y) - i\sin(y)$ . Then
- (A)  $f$  and  $g$  are differentiable on  $\mathbb{C}$
  - (B)  $f$  is differentiable on  $\mathbb{C}$  but  $g$  is not differentiable on  $\mathbb{C}$
  - (C)  $f$  is not differentiable on  $\mathbb{C}$  but  $g$  is differentiable on  $\mathbb{C}$
  - (D) Both  $f$  and  $g$  are nowhere differentiable on  $\mathbb{C}$
22. Suppose  $\left| \int_{|z|=1} e^{1/z} dz \right| \leq M$ , then what can you say about  $M$  ?
- (A)  $M = 2\pi$
  - (B)  $M = 2\pi e$
  - (C)  $M = \frac{1}{2\pi}$
  - (D)  $M = \frac{1}{2\pi e}$
23. If  $\gamma$  is a positively oriented circle  $|z - 2i| = 4$ . Then what is  $\int_{\gamma} \frac{zdz}{z^2 + 9}$  ?
- (A)  $2\pi i$
  - (B)  $2\pi$
  - (C)  $\pi i$
  - (D)  $-\pi i$
24. Let  $a \in \mathbb{C}$ . Consider the power series  $\sum_{k=0}^{\infty} a^{k^2} z^k$ . What is the radius of convergence  $R$  of this series ?
- (A)  $R = 1$  for all  $a \in \mathbb{C}$
  - (B)  $R = \infty$  for  $|a| > 1$ ,  $R = 0$  for  $|a| < 1$  and  $R = 1$  if  $|a| = 1$
  - (C)  $R = 0$  if  $|a| > 1$ ,  $R = \infty$  if  $|a| < 1$  and  $R = 1$  if  $|a| = 1$
  - (D)  $R = \infty$  for all  $a \in \mathbb{C}$



25. Let  $J_n$  be the set of the congruence classes mod  $n$ . Consider the two statements :

1.  $n$  is prime.

2. In  $J_n$   $[a][b] = [0]$  implies that  $[a] = [0]$  or  $[b] = [0]$ . Then

(A)  $1 \Rightarrow 2$  but  $2 \not\Rightarrow 1$

(B)  $1 \not\Rightarrow 2$  but  $2 \Rightarrow 1$

(C)  $1 \not\Rightarrow 2$  and  $2 \not\Rightarrow 1$

(D)  $1 \Rightarrow 2$  and  $2 \Rightarrow 1$

26. Which of the following are groups under the respective operations ?

1.  $G_1 = \{1, 2, 3\}$  under multiplication modulo 4.

2.  $G_2 = \{1, 2, 3, 4\}$  under multiplication modulo 5.

(A) Only  $G_1$

(B) Only  $G_2$

(C) Both  $G_1$  and  $G_2$

(D) None of  $G_1$  and  $G_2$

27. Let  $A_4$  be a group of all even permutations on the set  $\{1, 2, 3, 4\}$  and let  $H = \{(1), (12), (34)\}$ . Then

(A)  $H$  is not a subgroup of  $A_4$

(B)  $H$  is a subgroup of  $A_4$  but not a normal subgroup of  $A_4$

(C)  $H$  is a normal subgroup of  $A_4$

(D)  $H$  is a subgroup of  $A_4$  but not a cyclic subgroup of  $A_4$

28. Suppose  $R$  and  $S$  are commutative rings with unities. Let  $\phi : R \rightarrow S$  be a ring homomorphism and let  $I$  be a prime ideal of  $S$  and let  $\phi^{-1}(I) = \{x \in R \mid \phi(x) \in I\}$ . Then

(A)  $\phi^{-1}(I)$  need not be an ideal of  $R$

(B)  $\phi^{-1}(I)$  is an ideal of  $R$  but not a prime ideal of  $R$

(C)  $\phi^{-1}(I)$  is a prime ideal of  $R$

(D)  $\phi^{-1}(I)$  is a maximal ideal of  $R$



29. Which of the following statements are true ?

1. There is no integral domain with exactly 6 elements.
2. Every finite integral domain is a field.

(A) Both 1 and 2 are true

(B) Only 1 is true

(C) Only 2 is true

(D) None of 1 and 2 is true

30. The polynomial  $f(x) = 2x^2 + 4$  is

(A) Reducible over  $\mathbb{Z}$  as well as over  $\mathbb{Q}$

(B) Irreducible over  $\mathbb{Z}$  as well as over  $\mathbb{Q}$

(C) Reducible over  $\mathbb{Q}$  but not on  $\mathbb{Z}$

(D) Reducible over  $\mathbb{Z}$  but not reducible over  $\mathbb{Q}$

31. Consider the Initial Value Problem (IVP)  $\frac{dy}{dx} = y^2$ ,  $y(0) = 1$  on which of the interval solution to the above IVP exists at all points ?

(A)  $(-\infty, \infty)$

(B)  $[-2, 2]$

(C)  $(-\infty, 1)$

(D)  $(0, \infty)$

32. Consider a system of equations :

$$y_1' = y_1$$

$$y_2' = -y_2$$

then which of the following represents the saddle point to this system ?

(A)  $(1, 0)$

(B)  $(0, 1)$

(C)  $(0, 0)$

(D)  $(1, 1)$



33. Which of the following represents singular solution of  $y = x \frac{dy}{dx} + \left( \frac{dy}{dx} \right)^2$  ?

(A)  $y = \frac{-x^2}{4}$

(B)  $y = x^2$

(C)  $y = \frac{x^2}{2}$

(D)  $y = \frac{x}{2}$

34. Consider  $d\left(\tan^{-1}\left(\frac{y}{x}\right)\right) = 0$ , this differential represents which of the following differential equations ?

(A)  $x dx + y dy = 0$

(B)  $-y dx + x dy = 0$

(C)  $y dx + x dy = 0$

(D)  $x dx - y dy = 0$

35. Which of the following is solution of  $\frac{d^2y}{dx^2} - y = \sin x$  ?

(A)  $y = C_1 e^{-2x} + C_2 e^{-x}$

(B)  $y = C_1 e^{-x} + C_2 e^x + \log x$

(C)  $y = C_1 e^{-x} + C_2 e^x - \frac{1}{2} \sin x$

(D)  $y = C_1 e^{-x} + C_2 e^x + x$

36. Which of the following is particular integral of  $x^3 y''' - 3x^2 y'' + 6xy' - 6y = x^4 \ln x$  ?

(A)  $y_p = \frac{x^4}{6} \left( \ln x - \frac{11}{6} \right)$

(B)  $y_p = \frac{x^3}{3} \left( \ln x - \frac{11}{6} \right)$

(C)  $y_p = \frac{x^2}{2} \left( \ln x - \frac{11}{6} \right)$

(D)  $y_p = \frac{x^4}{6} - \frac{11}{6}$

37. Consider one dimensional heat equation  $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ ,  $0 < x < \pi$ ,  $t > 0$  with boundary conditions  $u(0, t) = 0$ ,  $u(\pi, t) = 0$  and initial condition  $u(x, 0) = \sin x$ , which of the following represents solution  $u(x, t)$  ?

(A)  $u(x, t) = \sin x$

(B)  $u(x, t) = e^{-\alpha^2 t} \sin x$

(C)  $u(x, t) = e^{\alpha^2 t} \sin x$

(D)  $u(x, t) = \alpha^2 e^{-t} \sin x$



38. Consider  $(1 + y^2)u_{xx} + 2xy u_{xy} + (1 + x^2)u_{yy} + \sin(xy)u = 0$ . Then at point (1, 2) this equation is

- (A) Elliptic (B) Parabolic  
(C) Hyperbolic (D) Pair of lines

39. Which of the following is solution of  $\frac{\partial^2 z}{\partial x^2} + 4 \frac{\partial^2 z}{\partial x \partial y} - 5 \frac{\partial^2 z}{\partial y^2} = 0$  ?

(where  $f_1, f_2$  are any arbitrary differentiable functions of its variables)

- (A)  $z = f_1(y + x) + f_2(y - 5x)$  (B)  $z = f_1(y - x) + f_2(y - 2x)$   
(C)  $z = f_1(y + x) + f_2(y + 5x)$  (D)  $z = f_1\left(\frac{y+x}{y+5x}\right)$

40. Which of the following is solution of  $\left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right]y = \left(\frac{\partial z}{\partial y}\right)z$  ?

- (A)  $z - a^2y^2 = ax + b$  (B)  $z^2 - a^2y^2 = ax + b$   
(C)  $z^2 - ay = (ax + b)^2$  (D)  $z^2 - a^2y^2 = (ax + b)^2$

41. Which of the following is solution of  $(mz - ny)\frac{\partial z}{\partial x} + (nx - lx)\frac{\partial z}{\partial y} = ly - mx$  ?

(where 'f' is any arbitrary differentiable function of its variables)

- (A)  $x + y + z = f(lx + my + nz)$  (B)  $x^2 + y^2 + z^2 = f(lx + my + nz)$   
(C)  $x^2 + y^2 + z^2 = f(x + y + z)$  (D)  $x + y + z = f(l + m + n)$

42. Which of the following correctly lists the order of convergence of the numerical methods in increasing order ?

- (A) Bisection < Secant < Newton-Raphson  
(B) Secant < Bisection < Newton-Raphson  
(C) Bisection < Newton-Raphson < Secant  
(D) Newton-Raphson < Bisection < Secant



43. Suppose we use the secant method to approximate the value of  $\sqrt{2}$  using the equation  $x^2 - 2 = 0$  starting with  $x_0 = 1.5$  and  $x_1 = 1.4$ . Then the next iteration gives approximately  $x_2 = 1.4176$ . If the actual root correct upto 4 decimal places is 1.4142, then the absolute error and the percentage error at  $x_2$  are approximately
- (A) 0.0034 and 0.24% respectively      (B) 0.0026 and 0.19% respectively  
(C) 0.0015 and 0.10% respectively      (D) 0.0042 and 0.30% respectively

44. Consider the system of linear equations

$$100x_1 - 20x_2 - 30x_3 = 1$$

$$-20x_1 + 100x_2 - 30x_3 = 1$$

$$-20x_1 - 30x_2 + 100x_3 = 1$$

If a sequence  $x^{(k)}$  of approximation to the solution  $x^*$  of the above system is obtained

by the Gauss-Seidel method starting with  $x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ .

Then  $x^{(1)} =$  \_\_\_\_\_ approximately.

(A)  $\begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \end{bmatrix}$

(B)  $\begin{bmatrix} 0.01 \\ 0.012 \\ 0.0156 \end{bmatrix}$

(C)  $\begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$

(D)  $\begin{bmatrix} 0.03 \\ 0.03 \\ 0.03 \end{bmatrix}$

45. Given the following values of  $y = f(x)$  at points  $x_0 = -1$ ,  $x_1 = 0$ ,  $x_2 = 1$ ,  $x_3 = 2$  and  $x_4 = 3$ , the values of  $y = f(x)$  are  $y_0 = f(x_0) = 9$ ,  $y_1 = f(x_1) = 5$ ,  $y_2 = f(x_2) = 3$ ,  $y_3 = f(x_3) = ?$  and  $y_4 = f(x_4) = 5$ . Then what will be the missing value using the Newton's forward difference interpolation polynomial ?
- (A) 5                      (B) 2                      (C) 4                      (D) 3

46. Let  $f(x) = \ln(x)$ . Suppose we approximate the definite integral  $\int_1^2 f(x) dx$  using the Trapezoidal rule with  $n = 4$  subintervals, then what is the maximum error bound of this approximation ?

(A)  $\approx \frac{1}{192}$

(B)  $\approx \frac{1}{48}$

(C)  $\approx \frac{1}{24}$

(D)  $\approx \frac{1}{12}$



47. Consider the initial value problem  $\frac{dy}{dx} = x^2 + y^2$  with  $y(0) = 1$ . If we use the Euler's explicit method with step-size  $h = 0.1$  to approximate  $y(0.2)$ , then what is its approximate value ?
- (A) 1.242 (B) 1.232  
(C) 1.222 (D) 1.210
48. The variation  $\delta I$  of a functional  $J(y) = \int_a^b F(x, y, y') dx$  is defined as
- (A) Total differential of  $J$   
(B) First derivative of  $J$   
(C) Linear part of the change in  $J$  due to variation in  $y$   
(D) Higher order differential of  $J$
49. Which of the following equations is satisfied by  $\phi(x, y, z)$  for which the mean square value of magnitude of the gradient over a certain region  $R$  of space is minimum, when  $\phi$  is given on the boundary of  $R$  ?
- (A)  $\nabla^2 \phi = 0$  (B)  $\nabla \phi = 0$   
(C)  $\nabla \phi^2 = 0$  (D)  $\nabla^2 \phi^2 = 0$
50. For which of the following value of  $y$ ,  $I = \int_{x_1}^{x_2} (1 + y'^2)^{1/2} dx$  is minimum ?
- (A)  $y = Cx + D$  (B)  $y = Cx^2 + D$   
(C)  $y = Cx + Dx^2$  (D)  $y = x^2$
51. For  $F = y'^2$ , which of the following is solution of Euler's equation ?
- (A)  $y'' = a$  (B)  $y'' = 0$   
(C)  $y'' = ax$  (D)  $y'' = ax + b$



52. The initial value problem  $\frac{d^2y}{dx^2} + \lambda y = f(x)$ ,  $y(0) = 1$ ,  $y'(0) = 0$ , can be transformed to the integral equation

(A)  $y(x) = \lambda \int_0^x (\eta - x) y(\eta) d\eta$

(B)  $y(x) = 1 - \int_0^x (\eta - x) f(\eta) d\eta$

(C)  $y(x) = \lambda \int_0^x (\eta - x) y(\eta) d\eta + 1 - \int_0^x (\eta - x) f(\eta) d\eta$

(D)  $y(x) = 1 + \int_0^x y(\eta) d\eta$

53. Consider the integral equation  $y(x) = \lambda \int_0^1 (1 - 3x\eta) y(\eta) d\eta$ . Let the solution of this equation when  $\lambda = 2$  be denoted by  $y_1(x)$  and the solution when  $\lambda = -2$  be denoted by  $y_2(x)$ . Then  $y_1(x) =$  \_\_\_\_\_ and  $y_2(x) =$  \_\_\_\_\_.

(A)  $B(1 - 3x)$  and  $A(1 - x)$

(B)  $A(1 - x)$  and  $B(1 - 3x)$

(C)  $B(1 - 3x)$  and 0

(D) 0 and  $A(1 - x)$

54. Consider the integral equations :

$I_1 : y(x) = \frac{1}{2} + x - \int_0^x (x - t) y^2(t) dt$

$I_2 : y(x) = 1 + \lambda \int_0^x (x - t) y(t) dt + \int_0^x (x - t) g(t) dt$

Then

- (A)  $I_1$  is Volterra integral equation of second kind and  $I_2$  is Fredholm integral equation of second kind
- (B)  $I_1$  is Fredholm integral equation of second kind and  $I_2$  is Volterra integral equation of second kind
- (C) Both are Volterra integral equations of first kind
- (D) Both are Fredholm integral equations of first kind



55. The resolvent Kernel for the integral equation  $g(s) = f(s) + \lambda \int_{-1}^1 (st + s^2t^2)g(t)dt$  is

(A)  $\frac{st}{\left(1 - \frac{2}{3}\lambda\right)} + \frac{s^2t^2}{\left(1 - \frac{2}{3}\lambda\right)}$

(B)  $\frac{st}{\left(1 - \frac{2}{5}\lambda\right)} + \frac{s^2t^2}{\left(1 - \frac{2}{3}\lambda\right)}$

(C)  $\frac{st}{\left(1 - \frac{2}{5}\lambda\right)} + \frac{s^2t^2}{\left(1 - \frac{2}{5}\lambda\right)}$

(D)  $\frac{st}{\left(1 - \frac{2}{3}\lambda\right)} + \frac{s^2t^2}{\left(1 - \frac{2}{5}\lambda\right)}$

56. A particle of mass  $m$  is moving, then which of the following represents its kinetic energy in polar co-ordinate system ?

(A)  $T = \frac{1}{2}m(\dot{r}^2 + (r\dot{\theta})^2)$

(B)  $T = \frac{1}{2}m(\dot{r}^2 + (r\dot{\theta})^2)$

(C)  $T = \frac{1}{2}m(\dot{r} + r\dot{\theta})^2$

(D)  $T = \frac{1}{2}m(r + r\dot{\theta})^2$

57. If a Lagrangian for a certain system is given by  $L = \frac{m\dot{x}^2}{2} - ax^2 - bx - c$ , then which of the following represents Lagrange's equation of motion ?

(A)  $\ddot{x} = -\frac{(2ax + b)}{m}$

(B)  $\ddot{x} = \frac{2ax + b}{m}$

(C)  $\ddot{x} = -(2ax + b)$

(D)  $\ddot{x} = 2ax + b$

58. With the usual notation for matrix form of Hamilton's equation of motion, which of the following is true ?

(A)  $J_{i(j+n)} = -\delta_{ij}$

(B)  $J_{ij} = \delta_{ij}$

(C)  $J_{ij} = -\delta_{ij}$

(D)  $J_{i(j+n)} = \delta_{ij}$



59. For system of  $n$  particles, which of the following represents canonical transformation generated by  $F_1(q_i, Q_i, t)$ , where  $q, Q$  and  $H$  have their usual meaning ?

(A)  $p_i = \frac{\partial F_1}{\partial q_i}, P_i = \frac{-\partial F_1}{\partial Q_i}, K = H + \frac{\partial F_1}{\partial t}$

(B)  $p_i = \frac{\partial F_1}{\partial \dot{q}_i}, P_i = \frac{-\partial F_1}{\partial Q_i}, K = H + \frac{\partial F_1}{\partial t}$

(C)  $\dot{p}_i = \frac{\partial F_1}{\partial \dot{q}_i}, P_i = \frac{-\partial F_1}{\partial Q_i}, K = H + \frac{\partial F_1}{\partial t}$

(D)  $p_i = \frac{\partial F_1}{\partial q_i}, \dot{P}_i = \frac{-\partial F_1}{\partial Q_i}, K = H + \frac{\partial F_1}{\partial t}$

60. If the work done by force  $\vec{F}$  to move the particle of mass  $m$  from point 1 to point 2 is independent of path, then which of the following is NOT true ?

(A)  $\vec{F} = -\nabla V$ , where  $V$  is a potential

(B)  $\oint_C \vec{F} \cdot d\vec{r} = 0$

(C)  $\text{Curl } \vec{F} = \vec{0}$

(D)  $\text{Div } \vec{F} = 0$

61. The expected value of the number of tails in two tosses of a coin is

(A) 0

(B) 1

(C) 2

(D) 3

62. Suppose  $(X, Y)$  is a bivariate random variable, where

$$Y = \begin{cases} 1 & \text{with probability } \frac{1}{8} \\ 2 & \text{with probability } \frac{7}{8} \end{cases}$$

$$\text{and } X|Y = \begin{cases} 2Y & \text{with probability } \frac{3}{4} \\ 3Y & \text{with probability } \frac{1}{4} \end{cases}$$

What is the value of  $E[X|Y = 2]$  ?

(A)  $\frac{13}{4}$

(B)  $\frac{15}{4}$

(C)  $\frac{18}{4}$

(D)  $\frac{29}{4}$



63. Given a bivariate data as follows :

| X  | Y |
|----|---|
| 1  | 1 |
| 3  | 2 |
| 4  | 4 |
| 6  | 4 |
| 8  | 5 |
| 9  | 7 |
| 11 | 8 |
| 14 | 9 |

What is the correlation coefficient between X and Y ?

- (A) 0.932 (B) 0.952  
(C) 0.968 (D) 0.977
64. Given  $r_{12} = 0.82$ ,  $r_{13} = 0.77$  and  $r_{23} = 0.80$ , what is the value of  $r_{23.1}$  ?
- (A) 0.42 (B) 0.46  
(C) 0.49 (D) 0.52

65. Suppose that X and Y are independent random variables having the common density function :

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

What is the density function of the random variable  $X|Y$  ?

- (A)  $f_{X|Y}(a) = \frac{1}{a+1}, 0 < a < \infty$  (B)  $f_{X|Y}(a) = \frac{2}{a+1}, 0 < a < \infty$   
(C)  $f_{X|Y}(a) = \frac{2}{a-1}, 0 < a < \infty$  (D)  $f_{X|Y}(a) = \frac{1}{(a+1)^2}, 0 < a < \infty$
66. Suppose that the probability that an item produced by a certain machine will be defective is 0.1. What is the probability that a sample of 10 items will contain atmost one defective item assuming that the quality of successive item is independent ?
- (A) 0.736 (B) 0.692  
(C) 0.587 (D) 0.452



67. Let  $X_1, X_2, \dots, X_n$  be a random sample from  $N_p(\mu, \Sigma)$  population. Then to test  $H_0 : \mu = \mu_0$  against  $H_1 : \mu \neq \mu_0$  one probability of type - I error is obtained as

(A)  $P\left[T^2 < \frac{(n-1)p}{n} F_{p, n-p}(\alpha)\right]$       (B)  $P\left[T^2 > \frac{(n-1)p}{(n-p)} F_{n-1, n-p}(\alpha)\right]$   
(C)  $P\left[T^2 > \frac{(n-1)p}{(n-p)} F_{p, n-p}(\alpha)\right]$       (D)  $P\left[T^2 < \frac{(n-1)p}{(n-p)} F_{p, n-p}(\alpha)\right]$

68. Consider the following statements :

I. If  $X_n \xrightarrow{P} X$ , then  $\lim_{n \rightarrow \infty} P\{|X_n - X| < \varepsilon\} = 0$ , for  $\varepsilon > 0$

II. If  $X_n \xrightarrow{\text{as}} X$ , then  $P\{\lim_{n \rightarrow \infty} X_n = X\} = 1$

Which of the following are true ?

- (A) Only I is true  
(B) Only II is true  
(C) Both I and II are true  
(D) Neither I nor II are true
69. Suppose  $\{X_K\}$  are independent sequence of random variables with  $P\{X_K = \pm 2^K\} = \frac{1}{2}$ . Which of the following is correct ?

(A) The sequence obeys WLLN

(B)  $M_X(t) = \frac{1}{2}(e^{2^K t} + e^{-2^K t})$

(C)  $\text{Var}(S_n) / n^2 \rightarrow \infty$  as  $n \rightarrow \infty$

(D)  $\text{Var}(S_n) = \frac{3}{4}(4^n - 1)$

70. Let  $X \sim \text{Uni}(-a, a)$ . Determine  $a$  so that  $P(|X| < 1) = P(|X| > 2)$ .

(A) 1      (B) 2  
(C) 3      (D) 4



71. A bivariate normal (BVN) distribution is given as follows :

$$f(x, y) = C \exp \{ - [16(x - 2)^2 - 12(x - 2)(y + 3) + 9(y + 3)^2] / 216 \}$$

What is the value of correlation coefficient ?

- (A) 1 (B) 1/2  
(C) 1/3 (D) 1/4

72. In a One-way ANOVA model, if  $V_w$  is denoted as variation within treatments, then  $V_w/\sigma^2$  distributed as

(Assume there are 'a' treatments and 'b' replications)

- (A) F – distribution with  $(a - 1, b - 1)$  df  
(B) F – distribution with  $(a - 2, b - 2)$  df  
(C) Chi-square distribution with  $(ab - 1)$  df  
(D) Chi-square distribution with  $a(b - 1)$  df

73. Let Y be  $N(\mu, 1)$ , a normal variate and the conditional distribution of X given Y has exponential distributions with mean 2Y. Then the variance of X is

- (A)  $4 + 4\mu^2$  (B)  $6 + 4\mu^2$   
(C)  $8 + \mu^2$  (D)  $8 + 4\mu^2$

74. An association study resulted in the following data :

|   |    |    |    |    |    |
|---|----|----|----|----|----|
| X | 2  | 3  | 5  | 1  | 8  |
| Y | 25 | 25 | 20 | 30 | 16 |

If a regression line of Y on X is to be fitted, then what is the slope of the line ?

- (A) 1.52 (B) -1.88  
(C) 1.2 (D) -1.68

75. Let X be a random variable having exponential distribution with mean  $\frac{1}{\lambda}$ ,  $\lambda > 0$ . Consider the prior distribution of  $\lambda$  as  $g(\lambda) = \lambda e^{-\lambda}$ ,  $\lambda > 0$ . Then based on a single observation on X, the Bayes estimate of  $\frac{1}{\lambda}$  under squared error loss function is

- (A)  $\frac{x+1}{2}$  (B)  $\frac{x-1}{2}$  (C)  $\frac{x+1}{3}$  (D)  $\frac{x-1}{3}$



76. Consider the following statements :

- I. Chebyshev's inequality is valid only for absolutely continuous distributions.
- II. Chebyshev's inequality is valid iff variance of the random variable exists.

Which of the following are correct ?

- (A) Only I is correct
- (B) Only II is correct
- (C) Both I and II are correct
- (D) Neither I nor II is correct

77. Let  $X$  and  $Y$  be independent binomial random variables having respective parameters  $(n, p)$  and  $(m, p)$ . What is the conditional probability mass function of  $X$  given  $X + Y = u$  ?

(A)  $\binom{m}{i} \binom{n}{u-i} / \binom{n+m}{u}$

(B)  $\binom{n}{i} \binom{m}{u} / \binom{m+n}{u}$

(C)  $\binom{n}{i} \binom{m}{u-i} / \binom{m+n}{u}$

(D)  $\binom{m}{u} \binom{n}{u-i} / \binom{m+n}{u}$

78. According to a survey reported by TOI (Times of India) on the implementation of NEP, it was found that 52 percent of the population was in favor of implementing the program with a margin of error of  $\pm 4$  percent. Approximately, how many people participated in the survey ?

- (A) 490
- (B) 530
- (C) 580
- (D) 600

79. A sample of 50 transistors is simultaneously put on test that is to be ended when the 15<sup>th</sup> failure occurs. If the total time on test of all transistors is equal to 525 hrs, then what is the 95<sup>th</sup> confidential interval for the mean lifetime of a transistor assuming that the underlying distribution is exponential ?

(Given that  $\chi^2_{.025, 30} = 46.98$ ,  $\chi^2_{.975, 30} = 16.89$ )

- (A) (22.35, 73.48)
- (B) (22.35, 62.17)
- (C) (18.79, 62.17)
- (D) (18.79, 73.48)



80. In a multiple linear regression model, the sum of squares of residual corresponding to  $n = 8$ ,  $k = 2$  is obtained as 34.12. What is the estimate of  $\sigma^2$  ?

(A) 3.35 (B) 4.58  
(C) 5.55 (D) 6.82

81. In a One-way ANOVA model  $X_{ij} \sim N(\mu_i, \sigma^2)$ ,  $i = 1, 2, \dots, m$ ,  $j = 1, 2, \dots, n$ . If  $H_0 : \mu_1 = \mu_2 = \dots = \mu_m$  is true, then what is the unbiased estimate of  $\sigma^2$  ?

( $SS_b$  : between sample sum of squares,  $SS_w$  : Within sample sum of squares)

(A)  $SS_b/(nm - 1)$  (B)  $SS_w/(nm - 1)$   
(C)  $SS_b/(m - 1)$  (D)  $SS_w/(m - 1)$

82. A study to carry out Gender distribution of startup projects of various funding agencies are given below :

| Agency<br>Gender | UGC | OST | CSIR | Total |
|------------------|-----|-----|------|-------|
| Male             | 68  | 56  | 32   | 156   |
| Female           | 52  | 72  | 20   | 144   |

What is the test statistic value if gender is independent of the funding agencies ?

(A) 5.58 (B) 6.43 (C) 7.12 (D) 8.19

83. The p-value corresponds to a sign-test is

(A)  $\min \left[ P \left\{ \text{Bin} \left( n, \frac{1}{2} \right) \leq v \right\}, P \left\{ \text{Bin} \left( n, \frac{1}{2} \right) \geq v \right\} \right]$

(B)  $\max \left[ P \left\{ \text{Bin} \left( n, \frac{1}{2} \right) \leq v \right\}, P \left\{ \text{Bin} \left( n, \frac{1}{2} \right) \geq v \right\} \right]$

(C)  $\min \left[ P \left\{ \text{Bin} \left( n, \frac{1}{2} \right) \leq 1 - v \right\}, P \left\{ \text{Bin} \left( n, \frac{1}{2} \right) \geq 1 - v \right\} \right]$

(D)  $\max \left[ P \left\{ \text{Bin} \left( n, \frac{1}{2} \right) \leq 1 - v \right\}, P \left\{ \text{Bin} \left( n, \frac{1}{2} \right) \geq 1 - v \right\} \right]$



84. A LPP is given as follows :

$$\text{Maximize } Z = 4x_1 + x_2 + 2x_3$$

$$\text{Subject to } x_1 + x_2 + x_3 \leq 1,$$

$$x_1 + x_2 - x_3 \leq 0,$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

What is the maximum value of the function ?

(A) 1

(B) 2

(C) 3

(D) 4

85. The Poisson distribution with parameter  $\theta$  has pgf (probability generating function)

(A)  $e^{\theta(s+1)}$

(B)  $e^{\theta s - 1}$

(C)  $e^{1 - \theta s}$

(D)  $e^{\theta(s-1)}$

86. Given the transition probability matrix as

$$P = \begin{pmatrix} 0.75 & 0.25 & 0.00 \\ 0.25 & 0.50 & 0.25 \\ 0.00 & 0.75 & 0.25 \end{pmatrix}$$

What is the value of  $f_{12}^{(3)}$  if the state space is  $I = \{1, 2, 3\}$  ?

(A) 0.56

(B) 0.44

(C) 0.29

(D) 0.14

87. Consider the following statements :

I. A class of essential state form a minimal closed class.

II. A state  $i$  is transient iff  $\sum_{n=0}^{\infty} P_{ii}^{(n)} < \infty$ .

Which of the following are correct ?

(A) Only I is correct

(B) Only II is correct

(C) Both I and II are correct

(D) Neither I nor II is correct



88. Given the TPM as

$$P = \begin{pmatrix} 0.25 & 0.50 & 0.25 \\ 0 & 0 & 1 \\ 0.65 & 0.35 & 0 \end{pmatrix}$$

and the initial probabilities as  $P(X_0 = i) = \frac{1}{3}$ ,  $i = 1, 2, 3$ . What is the value of  $P(X_1 = 1)$  ?

- (A) 0.1 (B) 0.2  
(C) 0.3 (D) 0.4

89. Given a Markov chain with state space  $I = \{1, 2, 3\}$  and

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \end{pmatrix}$$

What is the value of mean recurrence time of state 2 ?

- (A) 0 (B) 1 (C) 2 (D) 3

90. If  $X(t)$  is a Poisson process, then the autocorrelation between  $X(2)$  and  $X(5)$  is

- (A) 0.15 (B) 0.20 (C) 0.32 (D) 0.65

91. For a queueing model :  $M/M/1(\infty)/FCFS$  model, the probability of  $n^{\text{th}}$  realization is obtained as

- (A)  $\rho(1 - \rho)^n$  (B)  $\rho^n(1 - \rho)$   
(C)  $\rho^{n-1}(1 - \rho)$  (D)  $\rho(1 - \rho)^{n-1}$

92. A Markov process for which  $X_n = \sum_{j=0}^{\infty} \alpha^j \Sigma_{n-j}$

- (A)  $V(X_n) \rightarrow \infty$  as  $n \rightarrow \infty$   
(B)  $\text{Cov}(X_n, X_k) \rightarrow 0$  as  $n \rightarrow \infty$   
(C)  $\text{Cov}(X_n, X_k) \rightarrow 0$  as  $n \rightarrow \infty$   
(D)  $\text{Cov}(X_n, X_k) \rightarrow \infty$  as  $n \rightarrow \infty$



93. The Cramer-Rao lower bound for  $e^{-\theta}$  in Poisson distribution with parameter  $\theta$  is

(A)  $\frac{n}{\theta} e^{-2\theta}$

(B)  $\frac{2\theta}{n} e^{-\theta}$

(C)  $\frac{\theta}{n} e^{-2\theta}$

(D)  $\frac{\theta^2}{n} e^{-2\theta}$

94. One of the best way to model the stock price of a company for a particular period is using

(A) Renewal process

(B) Yule process

(C) Brownian motion process

(D) Autoregressive process

95. A negative binomial distribution is defined by the pmf

$$p(Y = y) = \binom{y+r-1}{r-1} p^r q^y, y = 0, 1, 2, 3, \dots$$

$$0 \leq p, q \leq 1$$

If  $\mu = \frac{rq}{p}$  is the mean of the distribution, then what is the moment estimator of the parameter  $p$ ?

(A)  $\frac{r}{\mu}$

(B)  $\frac{r}{r+\mu}$

(C)  $\frac{\mu}{r+\mu}$

(D)  $\frac{\mu+r}{r}$

96. The efficiency of median relative to mean in normal distribution is

(A)  $\pi$

(B)  $\frac{\pi}{2}$

(C)  $\frac{2}{\pi}$

(D)  $\pi^2$

97. "The unbiased estimators based on complete sufficient statistics are unique". This result is due to

(A) Cramer-Rao

(B) Neyman

(C) Lehman-Scheffee's

(D) S. Basu



98. In some institution, the proportion of left handed student is 30% in a sample of 100. What will be the 95% confidence interval for the population proportion ?
- (A) (0.202, 0.8002)  
(B) (0.2101, 0.3898)  
(C) (0.3725, 0.4599)  
(D) (0.3826, 0.5325)
99. The Kendall's statistic corresponds to a sample of size 7 is 4. What is the estimator associated with the parameter ?
- (A) 0.190  
(B) 0.165  
(C) 0.178  
(D) 0.212
100. The relative efficiency of two non-parametric tests is the ratio of
- (A) Size of the two tests  
(B) Power of the two tests  
(C) Size of the two samples  
(D) Two medians
-



Space for Rough Work



Space for Rough Work

