MATHEMATICAL SCIENCES
Paper - II

Signature of Invigilators — Roll No. [ ][ ][ ][ ][ ]

(In figures as in Admit Card)

1. ....

Dec-08/01

Roll No. ....

2. ....

....

(in words)

Name of the Areas/Section (if any) ....

Time Allowed : 75 Minutes] **[Maximum Marks : 100**

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page.
2. This paper consists of **Seventy Four (74)** multiple choice type questions. Attempt any **fifty (50)** questions.
3. Each item has upto four alternative responses marked (A), (B), (C) and (D). The answer should be a capital letter for the selected option. The answer letter should entirely be contained within the corresponding square.

   Correct method [A] Wrong method [A OR [A]
4. Your responses to the items for this paper are to be indicated on the ICR Answer Sheet under Paper II only.
5. Read instructions given inside carefully.
6. Extra sheet is attached at the end of the booklet for rough work.
7. You should return the test booklet to the invigilator at the end of paper and should not carry any paper with you outside the examination hall.

પરીક્ષાર્થીઓ માટે સૂચનાઓ :

૧. આ પાનાની ટોચમાં દર્શાવેલી જગ્યામાં તમારો રોલનંબર લખો.

૨. આ પ્રશ્નપત્રમાં બહુવૈકલ્પિક ઉત્તરો ધરાવતા કુલ **ચુમ્મોતેર (૭૪)** પ્રશ્નો આપેલા છે. કોઈપણ **(૫૦)** પ્રશ્નોના જવાબ આપો.

૩. પ્રત્યેક પ્રશ્ન વધુમાં વધુ ચાર બહુવૈકલ્પિક ઉત્તરો ધરાવે છે. જે (A), (B), (C) અને (D) વડે દર્શાવવામાં આવ્યા છે. પ્રશ્નનો ઉત્તર કેપીટલ સંજ્ઞા વડે આપવાનો રહેશે. ઉત્તરની સંજ્ઞા આપેલ ખાનામાં બરાબર સમાઈ જાય તે રીતે લખવાની રહેશે.

   ખરી રીત : [A] ખોટી રીત : [A , [A]

૪. આ પ્રશ્નપત્રના જવાબ આપેલ ICR Answer Sheet ના Paper II વિભાગની નીચે આપેલ ખાનાઓમાં આપવાના રહેશે.

૫. અંદર આપેલ સૂચનાઓ કાળજીપૂર્વક વાંચો.

૬. આ બુકલેટની પાછળ આપેલું પાનું રફ કામ માટે છે.

૭. પરીક્ષા સમય પૂરો થઈ ગયા પછી આ બુકલેટ જે તે નિરીક્ષકને સોંપી દેવી. કોઈપણ કાગળ પરીક્ષા ખંડની બહાર લઈ જવો નહીં.

MATHEMATICAL SCIENCES

Paper–II

NOTE : This paper contains **SEVENTY FOUR** (74) multiple-choice/Assertion and Reasoning/Matching questions, each question carrying **two (2)** marks. Attempt **ANY FIFTY** questions.

નોંધ : આ પ્રશ્નપત્રમાં **ચુમ્મોતેર (૭૪)** બહુવિકલ્પીય પ્રશ્નો, સાચું-ખોટું અને જોડકાં બનાવવાના પ્રશ્નો છે. **પચાસ (૫૦)** પ્રશ્નોના જવાબ લખવાના છે. પ્રત્યેક પ્રશ્નના **બે (૨)** ગુણ છે.

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1. For the series $\sum_{n=1}^{\infty} \cos \frac{1}{n^3}$, which of the following is *true* ?

   (A) It is convergent but not absolutely convergent

   (B) It is absolutely convergent

   (C) It is divergent to ∞

   (D) It is not convergent but sequence of partial sums oscillates finitely

2. If f is a real-valued function uniformly continuous on (0, 1), then which of the following is *true* ?

   (A) f is a bounded function and would be continuous always

   (B) f is a continuous function always but may be unbounded

   (C) f may be a discontinuous function but would be bounded always

   (D) f may neither be bounded and may not be continuous

3. For the equation $1 + 2x + x^3 + 4x^5 = 0$, which of the following is *true* ?

   (A) It does not possess any real root

   (B) It possesses exactly one real root

   (C) It possesses exactly two real roots

   (D) It possesses exactly three real roots

4. If f and g are Riemann integrable functions such that $g \circ f$ is also defined. Then which of the following is *true* ?

(A) $g \circ f$ is always Riemann integrable

(B) If f is continuous and g is Riemann integrable, then $g \circ f$ is always Riemann integrable

(C) If g is continuous and f is Riemann integrable, then $g \circ f$ is always Riemann integrable

(D) $g \circ f$ is Riemann integrable only when one of them is a constant function

5. The domain of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{n^4}{(n+1)(n+2)} z^n \text{ is :}$$

(A) $\{z : |z| < 1\}$

(B) $\{z : |z| \leq 1\}$

(C) $\{0\}$

(D) $\{z : |z| < e\}$

6. If $r = \{z : |z| = 1\}$, then what is the value of :

$$\int_r \frac{\cos z}{z^2 (z-2)} dz \text{ ?}$$

(A) $-2\pi i$

(B) $-\frac{3\pi i}{2}$

(C) $-\frac{\pi i}{2}$

(D) 0

7. If $f(z) = e^{\frac{1}{z^2}}$, $z \neq 0$ and $f(0) = 0$ and $r = \{z : |z| = 1\}$, then which of the following is *true* ?

(A) f has a removable singularity at $z = 0$ and $\int_r f(z)\, dz = 0$

(B) f has a pole of order 2 at $z = 0$ and $\int_r f(z)\, dz = 0$

(C) f has an essential singularity at $z = 0$ and $\int_r f(z)\, dz = 0$

(D) $\int_r f(z)\, dz$ is indeterminate

8. Suppose B is a non-zero real skew-symmetric matrix of order 3 and A is a non-singular matrix with inverse C. Then rank of ABC is :

(A) 0, 1 or 2

(B) definitely 1

(C) definitely 2

(D) definitely 3

9. Let $U = \mathbf{R}^3$, be the vector space of 3 tuples of real numbers over $\mathbf{R}$. Let V be the subspace of U generated by the vectors (1, 2, –1), (4, –3, 2) and (1, –9, 5). Let W satisfy $U = V \oplus W$. Then dimension of W is :

(A) 0

(B) 1

(C) 2

(D) 3

10. The system of equations :

$$x + 2y + 4z = 5$$

$$-x + 3z = 6$$

$$3x + y - 2z = 7$$

possesses :

(A) No solution

(B) Unique solution

(C) Two solutions

(D) Infinite number of solutions

11. The real quadratic form X'AX, where

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{bmatrix}$$

is :

(A) Positive definite

(B) Negative definite

(C) Positive semidefinite

(D) Indefinite

12. If A is a square matrix of order $n > 1$ and if

$$p(\lambda) = a_0 \lambda^n + a_1 \lambda^{n-1} + \ldots\ldots + a_{n-1} \lambda + (a_n - 1) = 0$$

is its characteristic equation then a necessary and sufficient condition that 0 is an eigenvalue of A is :

(A) $a_n = 0$

(B) $a_n = 1$

(C) $a_n = 2$

(D) $a_n = n + 1$

13. Suppose that the distribution of a rv X is given by :

$$P(X = -1) = 1/6 = P(X = 4) \text{ and } P(X = 0) = 1/3 = P(X = 2).$$

Then find $E\left(\frac{X}{X+2}\right)$.

(A) $\frac{1}{36}$

(B) $\frac{1}{18}$

(C) $\frac{1}{9}$

(D) $\frac{1}{6}$

14. Suppose X_1 and X_2 are independent rvs with a common d.f. F. Let $X = \min(X_1, X_2)$. Then what is the df of X ?

(A) $F^2(x)$

(B) $1 - F^2(x)$

(C) $(1 - F(x))^2$

(D) $1 - (1 - F(x))^2$

15. The joint distribution of two rvs X and Y is given below :

X \ Y	1	2
0	$\frac{1}{4}$	$\frac{1}{4}$
1	$\frac{1}{4}$	$\frac{1}{4}$

Let $P_1 = \left\{\left(X < \frac{1}{2}\right), (Y > 1)\right\}$, $P_2 = \left\{\left(X \geq \frac{1}{2}\right), (Y > 1)\right\}$. Then which of the following holds ?

(A) The events in both P_1 and P_2 are independent

(B) Only in P_1 the events are independent

(C) Only in P_2 the events are independent

(D) In both P_1 and P_2 the events are not independent

16. Consider the functions :

$$F_1(x) = \begin{cases} e^x & , \text{ if } x < 0 \\ 1 & , \text{ if } x \geq 0 \end{cases}$$

and

$$F_2(x) = \begin{cases} 0 & , \text{ if } x < 0 \\ 1 - e^{-2x} & , \text{ if } x \geq 0 \end{cases}$$

Then which of the following is *true* ?

(A) Both F_1 and F_2 are d.fs.

(B) Only F_1 is a d.f.

(C) Only F_2 is a d.f.

(D) Neither F_1 nor F_2 is a d.f.

17. Let X and Y be two independent Poisson r.vs. with EX = EY. Let $\alpha = P(X \leq 2 | X + Y = 5)$. Then what can you say about α ?

(A) $\alpha < \frac{1}{2}$

(B) $\alpha > \frac{1}{2}$

(C) $\alpha = \frac{1}{2}$

(D) α cannot be evaluated because EX is not known

18. If P(A) = 0.63, P(B) = 0.5 and $P(A' \cap B) = 0.07$, then what is $P(A \cup B)$?

(A) 0.93

(B) 0.70

(C) 0.53

(D) 0.46

19. Consider an L.P.P. for which the constraints are given by the equation $A\underline{x} = \underline{b}$ ..... (*). Where A is an $m \times n$ matrix with $n > m$. Suppose $\underline{x}_0$ satisfies (*). Which of the following conditions will make $\underline{x}_0$ a basic feasible solution ?

(A) It contains exactly $(n - m)$ zeros

(B) It contains at most $(n - m)$ zeros

(C) It contains at least $(n - m)$ zeros

(D) It contains at least m zeros

20. Consider the following L.P. Problem :

Maximize $x + 3y + z$

Subject to $x + y \leq 3$

$x + z \leq 2$

$y + z \leq 3$

$x \geq 0, y \geq 0, z \geq 0.$

Then which of the following solutions is optimal ?

(A) $x = 1, y = 2, z = 1$

(B) $x = 1, y = 2, z = 0$

(C) $x = \frac{1}{2}, y = \frac{5}{2}, z = \frac{1}{2}$

(D) $x = 0, y_2 = 3, z = 0$

21. Consider the series $\sum_{n=4}^{\infty} \left(\frac{1}{n^2 - 1}\right)$. Which of the following is *true* ?

(A) The series converges and its sum is a rational number

(B) The series diverges to ∞

(C) The series oscillates

(D) The series converges and its sum is an irrational number

22. Let A be the set of all real algebraic numbers and B be the set of all sequences whose terms are members of {0, 3}. Then :

(A) both A and B are countable

(B) A is uncountable and B is countable

(C) A is countable and B is uncountable

(D) both A and B are uncountable

23. Let $f : [0, 2] \to \mathbf{R}$ be any monotonic function. Then which of the following is *true* ?

(A) f must be continuous everywhere, but f may not be differentiable everywhere

(B) f must be continuous everywhere except for a countable set

(C) f must be differentiable everywhere

(D) f need not be continuous at any point

24. Let $g_n(x) = \frac{nx}{1 + n^2x^2}$, $x \in \mathbf{R}$, $n = 1, 2, 3, ...$. Then the sequence $\{g_n\}$:

(A) converges pointwise everywhere and the limit function is continuous everywhere

(B) converges pointwise everywhere and the limit function is not continuous at some points

(C) converges pointwise only at $x = 0$

(D) converges pointwise at no $x \in \mathbf{R}$

25. Let $f(x) = \frac{1}{x}$, $x \neq 0$. Then which of the following is *true* ?

(A) $\lim_{x \to 0} f(x) = \infty$

(B) $\lim_{x \to 0} f(x) = 0$

(C) $\lim_{x \to 0} f(x) = 1$

(D) $\lim_{x \to 0} f(x)$ does not exist

26. If $\text{Arg}(z + 3) = \frac{\pi}{3}$, then the least value of $|z|$ is :

(A) 3

(B) $\frac{\sqrt{3}}{2}$

(C) $\sqrt{3}$

(D) $\frac{3\sqrt{3}}{2}$

27. The image of $\{z : \text{Re } z < 0, |\text{Im } z| < \pi\}$ under the exponential function is :

(A) interior of unit circle — $\{z : \text{Re } z = 0\}$

(B) lower half plane — $\{z : \text{Im } z = 0\}$

(C) interior of unit circle — $\{z : \text{Re } z \leq 0 \text{ and } \text{Im } z = 0\}$

(D) upper half plane — $\{z : \text{Im } z = 0\}$

28. The value of the cross ratio $(7 + i, 1, 0, \infty)$ is :

(A) $6 + i$

(B) $-6 + i$

(C) $6 - i$

(D) $-6 - i$

29. The function $f(x, y) = x^3 + ax^2y + bxy^2 + cy^3$ is entire only if :

(A) $a = 3i, b = -3, c = i$

(B) $a = 3i, b = -3, c = -i$

(C) $a = -3i, b = 3, c = i$

(D) $a = -3i, b = -3, c = i$

30. If $f(z) = \dfrac{z - 1 - i}{z^2 - (4 + 3i)\, z + (1 + 5i)}$, then which of the following is *true* ?

(A) all singularities of f are poles

(B) all singularities of f are removable

(C) f has a pole and a removable singularity

(D) f has no singularities

31. If H and K are subgroups of a group G and $S = H \cap K$, $T = H \cup K$, then which of the following is *true* ?

(A) Both S and T are subgroups of G

(B) Neither S nor T is a subgroup of G

(C) S is a subgroup of G but T may not be a subgroup of G

(D) T is a subgroup of G but S may not be a subgroup of G

32. Let H be a subgroup of a group G and $N(H) = \{g \in G \mid gHg^{-1} = H\}$. Then which of the following is *true* ?

(A) N(H) may not be a subgroup of G

(B) N(H) is a subgroup of G and H is a normal subgroup of N(H)

(C) N(H) is a subgroup of G but H may not be a subgroup of N(H)

(D) N(H) must be a normal subgroup of G

33. Let G be a group, A(G) be the group of all automorphisms of G under the binary operation of composition and

$$I(G) = \{T_g \in A(G) \mid g \in G,\ T_g(x) = g^{-1}xg,\ \forall x \in G\}.$$

Then which of the following is *true* ?

(A) I(G) is a subgroup of A(G), which may not be normal

(B) I(G) may not be a subgroup of A(G)

(C) I(G) must be an abelian subgroup of A(G)

(D) I(G) is always a normal subgroup of A(G)

34. Let U be an ideal of a ring R, $r(U) = \{x \in R \mid xu = 0 \text{ for all } u \in U\}$ and $[R : U] = \{x \in R \mid rx \in U \text{ for every } r \in R\}$. Then which of the following is *true* ?

(A) $r(U)$ and $[R : u]$ are both ideals of R

(B) $r(U)$ is an ideal of R but $[R : u]$ need not be an ideal of R

(C) $r(U)$ need not be an ideal of R but $[R : u]$ is an ideal of R

(D) none of $r(U)$ and $[R : u]$ may be an ideal of R

35. Let X be the set of all polynomials with real coefficients of degree 4 along with zero polynomial and Y be the set of all polynomials with real coefficients of degree ≤ 4 along with zero polynomial. Then which of the following is *true* ?

(A) X and Y are linear spaces over **R** and X is a linear subspace of Y

(B) X and Y are linear spaces over **R** and X is not a linear subspace of Y

(C) neither X nor Y is a linear space over **R**

(D) Y is a linear space over **R** but X is not a linear space over **R**

36. Let T : $\mathbf{R}^4 \to \mathbf{R}^4$ be a linear transformation. Then which of the following is *true* ?

(A) T must have some real eigenvalues which may be less than 4 in number

(B) T may not have any real eigenvalues at all

(C) T must have infinitely many real eigenvalues

(D) T must have exactly 4 real eigenvalues

37. Let $M = \begin{bmatrix} 2 & 0 \\ 5 & 1 \end{bmatrix}$. Then M satisfies the relation (I denotes 2 × 2 identity matrix) :

(A) $M^2 + 3M - 2I = 0$

(B) $2M^2 - 5M + I = 0$

(C) $M^2 - 3M + 2I = 0$

(D) $2M^2 + 5M - I = 0$

38. Let $T_1 : \mathbf{R}^3 \to \mathbf{R}^3$ be defined as $T_1(x, y, z) = (x, y + 1, 0)$ and $T_2 : \mathbf{R}^3 \to \mathbf{R}^3$ be defined as $T_2(x, y, z) = (x + y, y + z, z + x)$. Then which of the following is *true* ?

(A) Both T_1 and T_2 are linear transformations

(B) Neither T_1 nor T_2 is a linear transformation

(C) T_1 is a linear transformation but T_2 is not a linear transformation

(D) T_1 is not a linear transformation but T_2 is a linear transformation

39. As $x \to \infty$, every solution of

$$x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = 2, \ x > 0$$

approaches to :

(A) ∞

(B) 0

(C) 1

(D) $-\infty$

40. The bacteria in a certain culture increase at a rate proportional to the number present. The differential equation governing the process is :

(A) $\frac{dy}{dt} = ky$, where k is any real constant

(B) $\frac{dy}{dt} = k^2 y$,

(C) $\frac{dy}{dt} = y + kt$,

(D) $\frac{d^2y}{dt^2} = k^2 y$

41. For the differential equation $y \frac{dy}{dx} + x = 0$, which of the following is a solution ?

(A) $x^2 + y^2 = c$ (constant) and x is any real number

(B) $y = \sqrt{c - x^2}, -\sqrt{c} \le x \le \sqrt{c}$

(C) $y = -\sqrt{c - x^2}, -\sqrt{c} \le x \le \sqrt{c}$

(D) $y = \sqrt{c - x^2}, -\sqrt{c} < x < \sqrt{c}$

42. Which of the following is a solution of

$$\frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial^2 z}{\partial x \partial y} + 3 \frac{\partial^2 z}{\partial y^2} = 0 \ ?$$

(A) $z = \sin(y - 2x) + y \cos(y + x)$

(B) $z = x \sin(y + 2x) + \cos(y - x)$

(C) $z = \sin(y - 2x) + \cos(y - x)$

(D) $z = y \sin(y + 2x) + x \cos(y + x)$

43. Which of the following is a solution of :

$$\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0 \ ?$$

(A) $u = \exp(y - 2x)$

(B) $u = (2x + y)^3$

(C) $u = (y + 2x)\exp(y - 2x)$

(D) $u = (x + 2y)\exp(y - 2x)$

44. The partial differential equation having the family of spheres $(x - a)^2 + (y - b)^2 + z^2 = c^2$, where a, b, c are constants, as its integral surfaces will be :

(A) linear & first order

(B) non-linear & first order

(C) linear & second order

(D) non-linear & second order

45. The average of a group of 20 observations is 16. If 2 observations with average 10 are discarded and 6 observations with average 10 are included in the group, then what is the percentage change in the average of the group ?

(A) 0.0%

(B) 6.25% decrease

(C) 6.66% increase

(D) None of these

46. Let the line of regression of X on Y corresponding to a data set be $x = 2y + 4$. If $U = 2X + 4$, $V = 3Y - 4$, then find the coefficient of regression of U on V :

(A) 3/4

(B) 3

(C) 4/3

(D) 2

47. At a company employees must report at 7.30 a.m. The arrival times of employees follow normal with mean μ = 7.25 a.m. and standard deviation of 4 minutes. Find the probability that an employee arrives late. ($\Phi(x)$ denotes the distribution function of X ~ N(0, 1))

(A) $1 - \Phi\left(\frac{5}{4}\right)$ (B) $\Phi\left(\frac{5}{4}\right)$

(C) $\Phi\left(\frac{.05}{4}\right)$ (D) $1 - \Phi\left(\frac{.05}{4}\right)$

48. Let X and Y be iid exponential E(λ) variates. Then what is the distribution of $\frac{X}{X + Y}$?

(A) Beta (2, 1) of Type I (B) Beta (2, 1) of Type II

(C) Beta (1, 2) of Type I (D) Beta (1, 2) of Type II

49. Let $X_n \xrightarrow{p} X$ and $Y_n \xrightarrow{p} Y$. Then for $X_n + Y_n \xrightarrow{p} X + Y$ to hold, which of the following is appropriate ?

(A) only if both X and Y are degenerate

(B) only if both X_n and Y_n are independent, $\forall\ n \geq 1$

(C) only if X or Y is degenerate

(D) none of the above

50. For CLT to hold for a sequence $\{X_n\}$ of iid rvs, which of the following is appropriate ?

(A) It is necessary that $E\left(X_1^K\right) < \infty, \forall\, K \geq 1$

(B) It is sufficient that $E\left(X_1^2\right) < \infty$

(C) It is sufficient that characteristic function $\phi_{X_1}(t)$ exists

(D) It is sufficient that $E(X_1) = 0$

51. Let (X, Y) have density

$$f(x, y) = \begin{cases} \lambda_1 \lambda_2 e^{-\lambda_1 x - \lambda_2 y} & , \quad x > 0, y > 0 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Then find P(X < Y).

(A) $\frac{\lambda_2}{(\lambda_1 + \lambda_2)}$

(B) $\frac{\lambda_1}{(\lambda_1 + \lambda_2)}$

(C) $\lambda_1 \lambda_2$

(D) $\frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)}$

52. Let the rv X have density

$$f(x) = \begin{cases} 1 - |1 - x| & , \quad 0 < x < 2 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Then find $P\left(\frac{1}{2} \le X \le \frac{3}{2}\right)$.

(A) $\frac{3}{4}$

(B) $\frac{1}{2}$

(C) $\frac{1}{4}$

(D) $\frac{2}{3}$

53. Let $\{X_n\}_{n \ge 1}$ be a sequence of iid Bernoulli B(1, p) variates, and N ~ P(λ), Poisson with mean λ. Assume N is independent of $\{X_n\}$, and $S_0 \equiv 0$, degenerate at zero. Then what is the distribution of $S_N = X_1 + ... + X_N$?

(A) Binomial B(N, p)

(B) Poisson P(λp)

(C) Negative binomial NB(α, p)

(D) Geometric G(N)

54. Let X ~ U(0, θ) for θ > 0. Then which of the following holds ?

(A) V(X) < E(X)

(B) V(X) > E(X)

(C) V(X) = E(X)

(D) Nothing can be said in general

55. Let $X \sim B(n, p)$. For the distribution to have a unique mode, which of the following is appropriate ?

(A) np is an integer

(B) $(n + 1)p$ is not an integer

(C) $(n - 1)p$ is an integer

(D) $(n - 1)p$ and np are both integers

56. Let $X_1, ...X_n$ be iid N(0, 1) variables. Consider

$$Y_1 = \frac{\overline{X}\sqrt{(n-1)\,n}}{\sqrt{\sum_{1}^{n}(X_i - \overline{X})^2}} \text{ and } Y_2 = \frac{\sqrt{n}\,\overline{X}}{\sqrt{\sum_{1}^{n}(X_i - \overline{X})^2}}$$

Then which of the following is *true* ?

(A) Both Y_1 and Y_2 follow t_{n-1}

(B) Both Y_1 and Y_2 follow t_n

(C) $Y_1 \sim t_{n-1}$, but Y_2 does not follow t_n

(D) $Y_2 \sim t_n$, but Y_1 does not follow t_{n-1}

57. Let T_n and T'_n be two independent unbiased consistent estimators of a parameter θ. Then which of the following is an unbiased estimator of θ^2 ?

(A) T_n^2

(B) $T_n'^2$

(C) $T_n \cdot T_n'$

(D) None of these

58. Let $X_1, X_2, ..., X_n$ be a random sample from $N(\mu, 1)$ and let $T = \overline{X}^2 - \frac{1}{n}$ be an estimator of μ^2. Which of the following properties does T possess ?

(1) T is unbiased estimator of μ^2

(2) T attains Cramer-Rao lower bound of variance

(3) T is UMVUE of μ^2.

(A) (1) only

(B) (1) and (2) only

(C) (1) and (3) only

(D) (1), (2) and (3)

59. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from $N(0, \sigma^2)$. Find estimator of σ^2, obtained by the method of moments.

(A) $\frac{1}{n}\sum_{1}^{n} X_i$

(B) $\frac{1}{n}\sum_{i=1}^{n} X_i^2$

(C) $\frac{1}{n-1}\sum_{i=1}^{n} X_i^2$

(D) $\frac{1}{n-1}\sum_{i=1}^{n} (X_i - \overline{X})^2$

60. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from Poisson distribution $P(\lambda)$.

Let $T = \sum_{i=1}^{n} X_i$. Find UMVUE of $e^{-\lambda}$.

(A) $e^{-\frac{T}{n}}$

(B) $\left(1 - \frac{1}{n}\right)^{T}$

(C) $\left(1 - \frac{1}{n}\right)^{\frac{T}{n}}$

(D) None of these

61. Consider an exponential distribution with mean $\theta \in \{2, 3\}$. Consider the problem of testing $H_0 : \theta = 3$ *V/s* $H_1 : \theta = 2$. A random sample of size 5 is selected. If $T = \sum_{i=1}^{5} X_i$ is the sum of the observed values, what is the critical region of size α ?

(A) $T \geq C_0$ where C_0 is such that $P(T \geq C_0 | \lambda = 2) = \alpha$

(B) $T < C_0$ where C_0 is such that $P(T < C_0 | \lambda = 2) = \alpha$

(C) $T \geq C_0$ where C_0 is such that $P(T \geq C_0 | \lambda = 3) = \alpha$

(D) $T < C_0$ where C_0 is such that $P(T < C_0 | \lambda = 3) = \alpha$

62. Let $X_1, \ldots\ldots, X_n$ be iid $N(\theta, \sigma^2)$ $H_0 : \sigma^2 = 1$, $H_1 : \sigma^2 > 1$. Which of the following is the form of the acceptance region for H_0 for a suitable test ?

(A) $\sum X_i^2 < c$

(B) $\sum (X_i - \overline{X})^2 > c$

(C) $\sum (X_i - \overline{X})^2 < c$

(D) $|\overline{X}| < c$

63. Let the problem of interest be to test that observations have $u(0, 1)$ distribution. If observations are 0.82 and 0.31, which of the following is the value of the Kolmogorov-Smirnov statistic ?

(A) 0.32
(B) 0.51
(C) 0.82
(D) 0.31

64. Consider the problem of testing for x_0 to be the third quartile of a continuous distribution. Let T be the number of observations less than x_0 in a random sample of size n. What is the null distribution of T ?

(A) $B(n, 1/2)$
(B) $B(n, 1/4)$
(C) $B(n, 3/4)$
(D) None of these

65. Which of the following relations, given in standard notations for M/M/1 model, is *not true* ?

(A) $W_s = W_q + \frac{1}{\lambda}$
(B) $L_s = \lambda W_s$
(C) $L_s = L_q + \frac{1}{\lambda}$
(D) $L_q = \lambda W_q$

66. For a supplier's model with instantaneous production and without shortages, find Wilson-Harris lot size formula :

(A) $\sqrt{2DC_o/C_h}$
(B) $\sqrt{2C_o/DC_h}$
(C) $\sqrt{DC_o/C_h}$
(D) $\sqrt{C_o/DC_h}$

where D is demand per unit time

C_h is holding cost per unit time

C_o is ordering cost per order

67. For an M/M/1 queuing model with λ = 30 customers per day (24 hours) $\frac{1}{\mu}$ = 36 minutes, find the expected number of customers in the system.

(A) 5
(B) 4
(C) 3
(D) 1

68. The following is an optimal simplex tableau of an l.p.p. where X_1, X_2 are decision variables and X_3, X_4 are slack variables.

		c_j	2	4	0	0
Objective coefficients	Basic Variables	Soln.	X_1	X_2	X_3	X_4
4	X_2	5/2	1/2	1	1/2	0
0	X_1	3/2	1/2	0	–1/2	1
		$z_j - c_j$	0	0	2	0

Then which of the following is the right hand side vector of the original l.p.p. ?

(A) $\begin{pmatrix} 5/2 \\ 3/2 \end{pmatrix}$

(B) $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$

(C) $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$

(D) $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$

69. For a primal-dual pair with max z_1 as primal objective and min z_2 as dual objective, which of the following is always *true* ?

(A) $z_1 > z_2$

(B) $z_1 = z_2$

(C) $z_1 \leq z_2$

(D) $z_1 \geq z_2$

70. Consider a two-person zero-sum game, given by :

		Player B		
		B_1	B_2	B_3
	A_1	2	4	5
Player A	A_2	10	7	Q
	A_3	4	P	6

To have the saddle point at the entry (2, 2) which set of range of values of P and Q is *incorrect* ?

(A) P ≤ 7, Q > 7

(B) P ≤ 7, Q ≥ 7

(C) P < 7, Q ≥ 7

(D) P > 7, Q > 7

71. Two persons P_1 and P_2 draw simple random samples of size 3 from a population consisting of $\{U_1, U_2, U_3, U_4$ and $U_5\}$. The samples that they drew were :

$$P_1 : \{U_5, U_2, U_4\} \text{ and } P_1 : \{U_2, U_5, U_2\}.$$

Then which of the following statements is definitely *false* ?

(A) P_1 must have drawn by SRSWR

(B) P_2 must have drawn by SRSWR

(C) P_1 must have drawn by SRSWOR

(D) P_2 must have drawn by SRSWOR

72. In large samples, with SRS, what is a condition for the ratio estimate $\hat{Y}_R = \frac{\hat{Y}}{\hat{X}} . X$ to have a smaller variance than the estimate $\hat{Y} = N\bar{y}$?

(A) $2\rho > C_x/C_y$

(B) $2\rho < C_x/C_y$

(C) $2\rho < C_y/C_x$

(D) $2\rho > C_y/C_x$

Where C_x, C_y and ρ are respectively the coefficient of variation of x, y and the correlation coefficient between x and y.

73. Which of the following basic principles is suitable for estimation of the magnitude of experimental error ?

(A) Randomization

(B) Replication

(C) Local control

(D) All these three

74. The following is an ANOVA TABLE of an LSD with 6 missing values denoted by a to f.

Source	**d.f.**	**s.s.**	**m.s.s.**	F_c
Rows	4	68.00	17.00	
Columns	4	150.00	37.50	
Treatments	a	b	c	7.732
Error	12	128.00	d	
Total	e	f		

Then find the value of b.

(A) 4

(B) 330

(C) 676

(D) 82.50

ROUGH WORK